

**SOLIDS
AND LIQUIDS**

On Quantum Effects on the Surface of Solid Hydrogen

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Abstract—The low-frequency spectrum of hypothetical superfluidity on the free surface of a quantum crystal of hydrogen is determined. In the quantum-rough state of the surface, crystallization waves with a quadratic spectrum should propagate. In the atomically smooth state, the spectrum is linear. Crystallization waves propagating along elementary steps are also considered.

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Rapid variation in the shape of hydrogen crystals at a temperature of about 1.8 K was observed in [1]. Such a behavior was not confirmed in [2, 3]. However, the hydrogen surface at lower temperatures has not been investigated as yet, and the possibility of quantum effects in this case remains unclear, like for the interface between solid and liquid helium prior to the discovery of crystallization waves [4, 5].

Surface point defects in a quantum crystal are delocalized. Upon an increase in tunnel effects, zero-point defects and their superfluidity appear [6]. Surface superfluidity in hydrogen was considered in [7] in connection with observation [1]. Upon their further growth, quantum fluctuations lead to a quantum-rough state of the surface [4]. Here, we pay attention to the considerable difference in the oscillation spectra of the surface superfluid liquid in the atomically smooth and quantum-rough states of the free surface of crystals.

The equations for the dynamics of a superfluid liquid have the form (see [8], (139, 7))

$$\dot{\mathbf{v}}_s + \nabla \tilde{\mu}_s = 0, \quad (1)$$

where $\tilde{\mu}_s = \mu_s/m$, μ_s is the chemical potential of the superfluid liquid, m is the mass of a hydrogen molecule, and \mathbf{v}_s is the velocity of the superfluid component. The variation of μ_s is defined as

$$\delta\mu_s = \frac{\partial\mu_s}{\partial n_s} \delta n_s, \quad (2)$$

where n_s is the density of the superfluid component. The continuity equation for a superfluid 2D liquid on an atomically smooth surface has the form

$$\dot{n}_s + \text{div} \mathbf{j} = 0, \quad (3)$$

where \mathbf{j} is the surface flux of particles. Disregarding the diffusion of the normal component, we have

$$\mathbf{j} = n_s \mathbf{v}_s. \quad (4)$$

The chemical potential μ_s of a 2D superfluid liquid on an atomically smooth surface can be in equilibrium

with the bulk value only in the case of exchange of particles on linear surface defects (steps). In accordance with Eqs. (1)–(4), in the absence of steps, oscillations of the surface superfluid liquid on an atomically smooth face are characterized by the standard line spectrum.

In the quantum-rough state, chemical potential μ_s does not differ from the bulk value μ of the chemical potential. In this case, the part of μ_s depending on the variation of surface profile $\zeta(x, y)$ is defined by the Herring capillary correction [10]

$$\delta\mu = -\frac{\alpha}{n} \Delta \zeta, \quad (5)$$

where α is the surface energy (we disregard anisotropy to simplify the form of expressions), n is the number density of particles in the crystal, and Δ is the 2D Laplace operator.

According to Mullins [9], the continuity equation on an atomically rough surface also differs significantly from that for an atomically smooth surface:

$$n \dot{\zeta} + \text{div} \mathbf{j} = 0. \quad (6)$$

Using relations (1) and (4)–(6), we can derive the equation for the linear dynamics of the surface shape:

$$\ddot{\zeta} + \frac{n_s \alpha}{n^2 m} \Delta^2 \zeta = 0. \quad (7)$$

Thus, the oscillations of a superfluid 2D liquid on a quantum-rough surface have a quadratic spectrum and reduce to recrystallization waves. Taking into account gravitation correction $mg\zeta$, we obtain the spectrum of gravitation–capillary crystallization waves:

$$\omega = k \sqrt{\frac{n_s}{n} g \left(1 + \frac{\alpha k^2}{mng} \right)}.$$

In hydrogen, conventional Rayleigh elastic waves obviously propagate over atomically smooth, as well as over quantum-rough, surfaces irrespective of whether surface superfluidity exist. For example, as in the case

of a helium crystal–liquid interface, there is no need to take into account their interaction with additional waves discussed here to determine the spectra in view of a considerable difference in their velocities.

It should be noted that in the case of surface superfluidity on an atomically smooth face over a step, bending waves accompanied by 2D mass transfer on the upper and lower faces adjoining the step (1D recrystallization waves with spectrum $\omega \propto k^{3/2}$) should propagate.

The temperature of the possible transition to the 2D superfluid state depends on the orientation of the surface. Even in the most favorable case in which superfluidity appears for all orientations, the time required for the crystal shape to reach the equilibrium configuration at zero temperature can be substantial, because a supercurrent is expected only for a thickness on the order of the atomic size. For example, for a critical temperature equal to the sound velocity on the order of 10^5 cm/s, a crystal with a size on the order of a centimeter may noticeably change its shape during a few minutes. However, the critical velocity is in all probability lower, and the effective “thickness” of the surface superfluid liquid is slightly smaller. Nevertheless, even days of shape relaxation at a certain temperature below the boundary of 1.8 K of the studied range and reduction in this time upon a further decrease in temperature would indicate surface superfluidity.

It is hardly possible to observe crystallization waves or waves on atomically smooth faces using optical methods as in the case the crystal–liquid interface in helium [5, 11]. Experiments on observing such additional branches must apparently be carried out by combining the method for studying surface diffusion on hydrogen crystal films [12] with measuring the spectrum of capillary waves in superfluid helium films [13].

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