Magnon correction to the resonance frequency in antiferromagnets

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The temperature correction to the resonance frequency in the antiferromagnetic material in the spin-wave region has been calculated. The theory is found to be in quantitative agreement with the experimental data for solid He^3.

The problem of finding the temperature corrections to the spin-wave spectrum in magnetic materials is generally linked with the high-spin approximation. Here the theory is rather cumbersome even in a very simple ferromagnetic case. The situation is particularly complicated in antiferromagnets in which the microscopic model cannot describe the ground state satisfactorily.

In the present experiments we have used a simple approach to determine the temperature corrections which is essentially similar to the approach developed by Andreev[1] in hydrodynamics and which is, in principle, not based upon model representations. The set of experimental data necessary for an exact testing (without any

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adjustable parameters) of the theory we were able to find for only one rather exotic magnetic material—solid antiferromagnetic He^3 (spin 1/2). For this reason (see Ref. 2), we will use a collinear antiferromagnet of the easy-plane type to carry out the calculations.

The low-frequency dynamics of such an antiferromagnet (at $T = 0$) is given by the equation[3]

$$[\mathbf{l}\ddot{\mathbf{l}}] = \frac{\gamma^2}{\chi_\perp}\left[\frac{\delta U}{\delta \mathbf{l}}\mathbf{l}\right], \tag{1}$$

where γ is the gyromagnetic ratio, $\chi_\perp$ is the susceptibility in the direction perpendicular to the antiferromagnetic vector l ($l^2 = 1$), and U is the potential energy

$$U = \frac{1}{2}\int dV\{\alpha_\parallel(\partial_z \mathbf{l})^2 + \alpha_\perp[(\partial_x \mathbf{l})^2 + (\partial_y \mathbf{l})^2] + \beta l_z^2\}. \tag{2}$$

Here $\alpha_\parallel$ and $\alpha_\perp$ are the exchange constants of the inhomogeneity energy, and β is the anisotropy constant. To streamline the calculations, we will analyze Eq. (1) in the dimensionless form

$$[\mathbf{l}\ddot{\mathbf{l}}] = \left[\frac{\delta U}{\delta \mathbf{l}}\mathbf{l}\right]; \tag{1'}$$

$$U = \frac{1}{2}\int\left\{(\partial \mathbf{l})^2 + l_z^2\right\} dV. \tag{2'}$$

Let us consider the small oscillations near the state $l_y = 1$. At a temperature $T \gg \hbar\omega(0)$, for which $\omega(0) = \gamma(\beta/\chi_\perp)^{1/2}$ (but which is low in comparison with a typical exchange energy), we can represent the amplitude of the motion l_z, l_x as a sum

$$l_z = \nu_z + \mu_z, \qquad l_x = \nu_x + \mu_x, \tag{3}$$

where the functions $(\nu_z, \nu_x) \equiv \boldsymbol{\nu}$ correspond to the relevant slow motion, and $(\mu_z, \mu_x) \equiv \boldsymbol{\mu}$ correspond to the fast thermal motion. For small values of $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ we have

$$l_y = \sqrt{1 - (\vec{\mu} + \vec{\nu})^2} \simeq 1 - (\vec{\mu} + \vec{\nu})^2/2. \tag{4}$$

We write out the x component of Eq. (1')

$$l_y \ddot{l}_z - l_z \ddot{l}_y = l_y \Delta l_z - l_z \Delta l_y - l_y l_z .$$

and we divide this equation by l_y

$$\ddot{l}_z - \frac{l_z \ddot{l}_y}{l_y} = \Delta l_z - \frac{l_z \Delta l_y}{l_y} - l_z .$$

Substituting expressions (3) and (4) into this equation, we expand the resulting equation over $\boldsymbol{\mu}$ up to the quadratic terms and linearize it over $\boldsymbol{\nu}$. Averaging over the fast thermal fluctuations ($\langle \boldsymbol{\mu} \rangle = 0$), we find

$$\ddot{\nu}_z(1+\langle\mu_z^2\rangle)+\frac{1}{2}\nu_z\langle\partial_t^2\vec{\mu}^2\rangle-2\dot{\nu}_z\langle\mu_z\dot{\mu}_z\rangle+\nu_z\langle\mu_z\ddot{\mu}_z\rangle$$

$$=(1+\langle\mu_z^2\rangle)\Delta\nu_z+\frac{1}{2}\nu_z\langle\Delta\vec{\mu}^2\rangle-2\langle\mu_z\partial\mu_z\rangle\partial\nu_z+\nu_z\langle\mu_z\Delta\mu_z\rangle-\nu_z\,. \qquad (5)$$

Since all mean values are multiplied by the amplitude of slow motion, these values, ignoring the nonlinear effects, should be determined in the equilibrium state ($\boldsymbol{\nu}=0$). We can then set $\langle\mu_z\dot{\mu}_z\rangle$,$\langle\mu_z\partial\mu_z\rangle$,$\langle\Delta\boldsymbol{\mu}^2\rangle$, and $\langle\partial_t^2\boldsymbol{\mu}^2\rangle$ to zero, and Eq. (5) reduces to

$$(\ddot{\nu}_z-\Delta\nu_z)(1+\langle\mu_z^2\rangle)+\nu_z(1+\langle\mu_z,\ddot{\mu}_z-\Delta\mu_z\rangle=0 \qquad (6)$$

The function μ_z clearly satisfies the equation

$$\ddot{\mu}_z=\Delta\mu_z-\mu_z\ ,$$

from which it follows that

$$\langle\mu_z,\ddot{\mu}_z-\Delta\mu_z\rangle=-\langle\mu_z^2\rangle\ .$$

Dividing Eq. (6) by $(1+\langle\mu_z^2\rangle)$, we thus find

$$\ddot{\nu}_z=\Delta\nu_z-\nu_z(1-2\langle\mu_z^2\rangle)\,. \qquad (7)$$

We use the following line of reasoning to determine $\langle\mu_z^2\rangle$. The energy of motion μ_z is

$$\frac{1}{2}\int\{\dot{\mu}_z^2+(\partial\mu_z)^2\}\,dV \qquad (8)$$

In this equation the first term is the kinetic energy (see Ref. 3); the anisotropy is ignored in the potential energy. We expand μ_z in a Fourier series in momenta

$$\mu_z=\sum_{\mathbf{k}}\mu_{\mathbf{k}}e^{i\mathbf{kr}}\ ,\qquad \mu_{\mathbf{k}}=\mu^*_{-\mathbf{k}}\,.$$

On the one hand, the mean energy of the thermal motion will then be

$$\frac{1}{2}\sum_{\mathbf{k}}\langle|\dot{\mu}_{\mathbf{k}}|^2+k^2|\mu_{\mathbf{k}}^2|\rangle=\sum_{\mathbf{k}}k^2\langle|\mu_{\mathbf{k}}^2|\rangle$$

(oscillator). On the other hand, in the temperature region of interest to us, this energy obviously is $\Sigma_k\epsilon_k\,n_k$, where $\epsilon_k=k$ is the energy of a magnon with the momentum $\mathbf{k}$, and $n_k(\epsilon_k/T)$ is the Planckian magnon-distribution function. We therefore have

$$\langle\mu_z^2\rangle=\int\frac{1}{k}\,n_k\frac{d^3k}{(2\pi)^3}=\frac{T^2}{12}\ . \qquad (9)$$

Finally, returning to conventional units, we find from (7) and (9) the following expression for the temperature dependence of the resonance frequency

$$\omega_A(T) = \omega(0)\left(1 - \frac{\gamma^2 T^2}{12\hbar\chi_\perp c_\parallel c_\perp^2}\right), \tag{10}$$

where $c_\parallel$ and $c_\perp$ are the velocities of the spin waves: $c_\parallel = \gamma(\alpha_\parallel/\chi_\perp)^{1/2}$ and $c_\perp = \gamma(\alpha_\perp/\chi_\perp)^{1/2}$. There is no $\sim T^2$ correction to the spin-wave velocity. In determining the first nonvanishing correction to the velocity ($\sim T^4$) (see Ref. 4), it is necessary to take into account in the potential energy the following terms in the gradients. As can be seen from (4), the temperature dependence (10) (at $\mathbf{v} = 0$ it is clear that $\langle \mu_z^2 \rangle = \langle \mu_x^2 \rangle$) is the same as the temperature dependence of the modulus of the antiferromagnetism vector ($\equiv \langle l_y \rangle$) (or of the sublattice magnetization).

In the case of ferromagnets, an averaging of the Landau-Lifshitz equations over the thermal motion gives the following expression for the resonance frequency in the uniaxial case:

$$\omega_F(T) = \omega(0)\left(1 - \frac{\gamma\zeta(3/2)}{4\pi^{3/2}M}\left(\frac{T}{A}\right)^{3/2}\right),$$

consistent with the result of Ref. 5 obtained in a microscopic model for $S \gg 1$ [in the result of Ref. 5, Eq. (31.3.3), there are three misprints]. Here M is the magnetization, A is the average characteristic of the magnon spectrum $\epsilon_k = A_\parallel k_z^2 + A_\perp(k_x^2 + k_y^2)$, given by $A = A_\perp^{1/3} \cdot A_\parallel^{1/6}$, and ζ is the Riemann function.

Osheroff *et al.*[2] have shown that at a temperature below 1.03 mK solid He^3 is an easy-plane collinear antiferromagnet. In the spin-wave region, they derived an empirical equation for the temperature dependence of the resonance frequency in a zero field. We will write this equation in the form

$$\omega_{\exp}(T) = \omega(0)(1 - 0.23\,T^2)$$

(the temperature is given in mK). Having measured the entropy, Osheroff and Yu[6] found the average velocity of the spin waves to be (8.4 ± 0.4) cm/s ($\equiv c_\parallel c_\perp^2$). Finally, from the measurements of the magnetic susceptibility carried out by Prewitt and Goodkind[7] we find $\chi_\perp^{-1} \simeq 1.9 \times 10^5$. Substituting these data into (10) ($\gamma = 2.04 \times 10^4$ Hz/Oe), we find

$$\omega(T) \simeq \omega(0)[1 - (0{,}20 \pm 0{,}03)T^2].$$

To the best of our knowledge, this is the first case in which a relationship between the various physical characteristics of a magnetic material predicted by the spin-wave theory has been confirmed experimentally.

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