Theory of exchange symmetry

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Three new classes of spin order are shown to be possible. In two of the classes, a spin density $\langle \mathbf{S}(\mathbf{r}) \rangle$ arises only when relativistic effects are taken into account. In one class—scalar magnetic materials—the magnetic susceptibility is zero at a temperature of absolute zero.

When relativistic effects are ignored, the state of a magnetically ordered substance is characterized by the exchange symmetry.[1] Structures (magnetic materials) determined by the average microscopic spin density $\langle \mathbf{S}(\mathbf{r}) \rangle$ were studied in Refs. 1 and 2. Andreev and Grishchuk[3] pointed out a different possibility for the realization of a spontaneous violation of exchange invariance, in which the spin density is zero, and the order parameter is the correlation function $\langle S_i(\mathbf{r}_1) S_j(\mathbf{r}_2) \rangle$, which is anisotropic in spin space. The symmetry of such structures (spin nematic liquid crystals) under time reversal, $t \to -t$, is not violated. In principle, there can also be more-complex structures, in which a spontaneous violation of exchange invariance and of $t \to -t$ symmetry arises only in multipoint correlation functions. The structures which are described by even correlation functions (four-point, six-point, etc.) are analogous to the spin nematics studied in Ref. 3. The structures which are described by odd correlation functions, in contrast, are substantially different from both magnetic materials and spin nematic liquid crystals.

Let us consider, for example, the three-point correlation function

$$\langle S_i(\mathbf{r}_1) S_j(\mathbf{r}_2) S_k(\mathbf{r}_3) \rangle \equiv S_{ijk}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3).$$

In the simplest case, S_{ijk} reduces to a tensor which is completely antisymmetric in terms of spin indices:

$$S_{ijk} = \varphi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) e_{ijk}.$$

From the general requirement[1)] that all spin scalars constructed from an order parameter (e.g., $S_{ijk} S_{ijk}$) remain unchanged under transformations of the exchange crystal symmetry group G it follows that the function $\varphi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ transforms in accordance with some one-dimensional representation of group G. If the spin correlation functions of any rank remain isotropic in spin space, only the $t \to -t$ symmetry will be violated. Such a magnetic material is naturally called a "scalar."

We introduce the three unit vectors **n**, **l**, **m**, which lie in a common plane in spin space and which make angles of 120° from each other. We assume that the correlation function S_{ijk} is

$$S_{ijk} = \varphi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)(n_i n_j n_k + l_i l_j l_k + m_i m_j m_k), \tag{1}$$

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where the function φ transforms in accordance with a one-dimensional representation of group G. The exchange symmetry of such a structure (a tensor magnetic material) is determined by the following elements: $C_6^s R$, U_2^s, C, and $C^- R$, where C_6^s is a spin rotation through an angle $\pi/3$, U_2^s is a spin rotation through an angle π around the axis perpendicular to the C_6^s axis, R is the transformation $t \to -t$, and C and C^- are transformations of group G which respectively do not and do change the sign of the function φ.

In both scalar and tensor magnetic materials, since the symmetry $t \to -t$ is violated, the incorporation of relativistic effects leads to a finite value of the spin density, $\langle \mathbf{S}(\mathbf{r}) \rangle \propto (v/c)^2$ (more on this below). In such magnetic materials, in contrast with spin nematic liquid crystals, a weak ferromagnetism, a magnetoelectric effect, and a piezomagnetism are also possible by virtue of the magnetic symmetry.

We find the third type of violation of exchange invariance which is manifested in correlation functions by examining the possible coexistence of an ordinary order and a tensor order. For example, let us assume that in addition to correlation function (1), the vector $\langle \mathbf{S}(\mathbf{r}) \rangle$—which transforms in accordance with the same one-dimensional representation of group G as the function φ and which is oriented perpendicular to the vectors **n**, **l**, **m**—is nonzero. The exchange symmetry of such a magnetic material is determined by the elements G_3^s, C, and $C^- R$.

A theory of the low-frequency dynamics of tensor magnetic materials, like that for spin nematics,[3] can be constructed in complete analogy with the dynamics of magnetic materials.[2] Scalar magnetic materials are exceptional cases. Here there is no reason for the appearance of low-frequency degrees of freedom, since there has been no spontaneous violation of the rotational invariance of an exchange Hamiltonian.

As was shown in Ref. 2, exchange symmetry imposes important restrictions on the components of the magnetic susceptibility tensor. In particular, the axial symmetry of collinear magnetic materials and uniaxial spin nematics causes the longitudinal susceptibility to vanish at absolute zero. For the same reasons,[2] the vanishing of all the components of the susceptibility tensor is associated with the spherical symmetry of the spin space in scalar magnetic materials. At a nonzero temperature T the susceptibility results from the appearance of magnons; here we evidently have $\chi \propto \exp(-\epsilon_0/T)$ at $T \ll \epsilon_0$, where ϵ_0 is the gap in the magnon spectrum. At $T=0$, a nonzero susceptibility is found when relativistic effects are taken into account (cf. Ref. 4; problem 3 associated with §71). To the exchange Hamiltonian of the noninteracting magnons $\Sigma_{\mathbf{k}} \epsilon_{\mathbf{k}} a_{\mathbf{k}\alpha}^{+} a_{\mathbf{k}\alpha}$, where α is a spin index (for simplicity we are assuming that there is a single species of $S=1$ quasiparticles, which are degenerate in accordance with the isotropy of the spin space), we add terms of a relativistic nature:

$$\sum_{\mathbf{k}} d_{\mathbf{k}}^{\alpha} a_{\mathbf{k}\alpha} + d_{\mathbf{k}}^{\alpha *} a_{\mathbf{k}\alpha}^{+} + \beta_{\mathbf{k}}^{\alpha\beta} a_{\mathbf{k}\alpha} a_{-\mathbf{k}\beta} + \beta_{\mathbf{k}}^{\alpha\beta} a_{-\mathbf{k}\beta}^{+} a_{\mathbf{k}\alpha}^{+} + \gamma_{\mathbf{k}}^{\alpha\beta} a_{\mathbf{k}\alpha}^{+} a_{\mathbf{k}\beta}$$

$$+ \mu H (a_{\mathbf{k},1}^{+} a_{\mathbf{k},1} - a_{\mathbf{k},-1}^{+} a_{\mathbf{k},-1}).$$

Restrictions which follow from the magnetic symmetry of the state are imposed on the coefficients $d_{\mathbf{k}}^{\alpha} \beta_{\mathbf{k}}^{\alpha\beta}$ and $\gamma_{\mathbf{k}}^{\alpha\beta} = \gamma_{\mathbf{k}}^{\beta\alpha *} \propto (v/c)^2$. The linear terms, $\propto d$, lead to effects of the nature of a weak ferromagnetism. Scalar (and tensor) magnetic materials are therefore weak antiferromagnets or ferromagnets. The average spin of the magnetic atoms in them is proportional to $(v/c)^2$. The quadratic terms, $\propto\beta$, lift the spin degeneracy of the magnons after a diagonalization of the Hamiltonian and lead to a dependence of the ground-state energy of a scalar magnetic material on the external field; i.e., they lead to a nonzero susceptibility $\chi\sim\mu^2\beta^2/\epsilon_0^3\propto(v/c)^6$.

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[1)]In magnetic materials, one must check, in addition to the invariance of the convolution $S_i(\mathbf{r})S_i(\mathbf{r}')$, the invariance of the expression $\{e_{ijk}S_i(\mathbf{r}_1)S_j(\mathbf{r}_2)S_k(\mathbf{r}_3)\}^2$. For the representations discussed in Ref. 1, this condition introduces no new limitations.

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