Spin potential

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A macroscopic derivation of the equations of the spin dynamics of paramagnets is offered. Some additional terms should be incorporated in the Bloch equations. The boundary conditions at an interface between two paramagnets are determined.

Although the Bloch equations of the spin dynamics of paramagnets[1] give a fairly good description of experimental data, they are somewhat unsatisfactory from the theoretical standpoint: The literature reveals no common, systematic derivation of all the terms in these equations on the basis of a macroscopic theory. It is thus not clear whether all possible effects have been taken into account. Also lacking are boundary conditions for these equations. Our purpose in the present paper is to bridge this gap.

We first consider a paramagnet in which the exchange forces are considerably stronger than the relativistic forces—the spin–orbit and magnetic dipole–dipole forces. If we completely ignore relativistic effects, and if there is no external field, the spin-dynamics equations should express the conservation of each projection of the total spin angular momentum:

$$\dot{S}^\alpha+\partial_k J_k^\alpha=0, \tag{1}$$

where S^α is the density of the α projection of the spin, and J_k^α is the flux density of the α component of the spin along direction k. In a paramagnet this flux occurs by spin diffusion. At small deviations of the paramagnet from equilibrium, this flux can be written as follows, in complete analogy with ordinary diffusion:

$$J_k^\alpha=-A\partial_k\mu^\alpha, \tag{2}$$

where A is a constant (we are not ignoring the crystallographic anisotropy), and the quantity μ^α is a spin analog of the chemical potential of particles. We call the quantity $\vec{\mu}=\{\mu^\alpha\}$ the "spin potential of the paramagnons" or simply the "spin potential." The behavior of the free energy of the paramagnet as a function of the spin density reduces to the following term at small deviations from equilibrium:

$$\frac{\gamma^2\mathbf{S}^2}{2\chi}, \tag{3}$$

where γ is the spin gyromagnetic ratio, and χ is the magnetic (spin) susceptibility of the paramagnet. By definition, the spin potential is equal to the derivative of energy (3) with respect to **S**:

$$\vec{\mu}=\frac{\gamma^2}{\chi}\mathbf{S}. \tag{4}$$

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Using (2) and (4), we can thus rewrite Eq. (1) in the customary form

$$\dot{\mathbf{S}} - D\Delta\mathbf{S} = 0, \tag{5}$$

where D is a spin diffusion coefficient ($D = A\gamma^2/\chi$).

At the interface between two exchange-coupled paramagnets, e.g., at an interface between crystalline and liquid ^{3}He or at an interface between two electron paramagnets, the conditions requiring the continuity of the normal component of the flux of each spin projection hold:

$$J^{\alpha}_{n1} = J^{\alpha}_{n2}. \tag{6}$$

These conditions, however, are not sufficient for solving Eqs. (5), which are second-order equations involving spatial derivatives. It is clear from this analogy that we must have the following equality of the spin potentials:

$$\vec{\mu}_1 = \vec{\mu}_2. \tag{7}$$

In the case of ordinary diffusion, the elementary event is a transport of a particle across an interface, while in our case it is a simultaneous change in the spin projection by $+1$ in one of the phases and by -1 in the other. If, on the other hand, the exchange coupling between the paramagnets is weakened by some factor, it may be necessary to replace (7) by the more general condition

$$J^{\alpha}_{n1} = K(\mu^{\alpha}_1 - \mu^{\alpha}_2), \tag{8}$$

where K is a spin kinetic coefficient, which should be particularly small in this case.

We now consider the terms which arise in Eq. (1) when relativistic effects and a magnetic field are taken into account. We assume that the field is weak in comparison with the characteristic field at which a spin polarization of the paramagnet comparable to the total polarization arises. Treating the corrections to Eq. (1) as the result of an expansion in the small field $\mathbf{H}$, in $\dot{\mathbf{H}}$, and in the small quantity $\vec{\mu}$ (measures of the deviation of the system from equilibrium), and retaining the leading terms of this expansion, we find

$$\dot{S}^{\alpha} + \partial_k J^{\alpha}_k = -\frac{\mu^{\alpha}}{\tau} + B e^{\alpha\beta\gamma} H^{\beta} \mu^{\gamma} + C\dot{H}^{\alpha}, \tag{9}$$

where τ, B, and C are constants. For the spin potential, using the increment $-\mathbf{MH}$, where $\mathbf{M} = \gamma\mathbf{S}$ is the magnetization, in energy (4), we find the following result:

$$\vec{\mu} = \frac{\gamma^2}{\chi}\mathbf{S} - \gamma\mathbf{H}. \tag{10}$$

In the standard notation, Eq. (9) becomes

$$\dot{\mathbf{M}} - D\Delta(\mathbf{M} - \chi\mathbf{H}) = -\frac{\mathbf{M} - \chi\mathbf{H}}{\tau} + \tilde{\gamma}\,[\mathbf{HM}] + \chi_{\infty}\dot{\mathbf{H}}, \tag{11}$$

where $\tilde{\gamma}=B\gamma/\chi$, and $\chi_\infty=C\gamma$. Equations (11) differ from the familiar spin-dynamics equations by virtue of their last term, which is proportional to the rate of change of the magnetic field. The quantity χ_∞ is evidently the high-frequency ($\omega\tau\gg 1$) magnetic susceptibility.

Without going into a microscopic theory, we cannot link the quantities γ and $\tilde{\gamma}$ with each other or with the spin gyromagnetic ratio of the free particles. In this regard the situation here is the same as in the dynamic theory of spin-ordered media.

In paramagnets in which relativistic effects are not small in comparison with exchange effects, the spin-dynamics equations differ from Eqs. (11) only in that there is no point in retaining the diffusion term in them. The reason is that a natural estimate of the diffusion coefficient yields $D\sim a^2/\tau$, where a is on the order of the distance between particles having spins. We can also do without boundary conditions in this case.

A special situation arises at an interface between paramagnets in one of which exchange effects are governing, while in the other they are not. For the first paramagnet, we then need boundary conditions, but they cannot be simply the purely exchange boundary conditions in (7) or (8). Conditions (6), on the other hand, are meaningless here, since the spin is not conserved in the second paramagnet. We should evidently replace condition (7) by a linear relationship between spin potentials (in such a manner that the boundary conditions are automatically satisfied at equilibrium):

$$\vec{\mu}_1=\eta\vec{\mu}_2+\rho\mathbf{n}(\mathbf{n}\mu_2), \tag{12}$$

where η and ρ are constants, and $\mathbf{n}$ is the unit vector normal to the interface. Conditions (8), on the other hand, become

$$J^\alpha_{n1}=\kappa\mu^\alpha_1+\nu n^\alpha(\mathbf{n}\vec{\mu}_1)+\eta\mu^\alpha_2+\rho n^\alpha(\mathbf{n}\vec{\mu}_2), \tag{13}$$

where κ, ν, η, and ρ are constants.

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[1] A. Abragam, *The Principles of Nuclear Magnetism*, Clarendon Press, Oxford, 1961.

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