Spin rigidity

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The vanishing of the longitudinal susceptibility in exchange spin-ordered structures with axial symmetry is explained. The effect stems from a more general property: spin rigidity. Two more types of axial structures—in addition to the three which have been mentioned previously in the literature—are possible.

As the temperature is lowered, the longitudinal magnetic susceptibility of insulating ferromagnets and collinear antiferromagnets is observed to decrease significantly. In the ferromagnets this behavior is quite natural: At absolute zero all spins have their maximum projection onto the selected axis, and the application of a magnetic field along this axis does not alter the state. A nonzero longitudinal susceptibility results from relativistic effects which tend to lower the degree of polarization in the ground state (§71 in Ref. 1).

No exact microscopic calculation has been carried out for antiferromagnets (the ground-state problem). An approximate (in terms of $1/S$) account of the effect of quantum fluctuations leads to a zero longitudinal susceptibility.[2] Since this assertion exists in the microscopic theory only as a result of calculations, the question of whether the susceptibility arises in some order in $1/S$ (or e^{-S}) remains open.

Andreev and the present author[3] have suggested that the absence of a longitudinal susceptibility from collinear antiferromagnets at absolute zero is a consequence of the axial symmetry of the ground state. The derivation of spin-dynamics equations and the assertion that the longitudinal susceptibility is zero, found in the process, are correct to within quadratic effects in the field. The nonlinear susceptibility thus remains an open question.

We show below that an even stronger assertion is correct: The ground-state energy of axial antiferromagnets is totally independent (if relativistic effects are ignored) of the longitudinal field, and the energy of spin excitations changes linearly in the field, with a universal proportionality factor, for any quasimomenta. This rigid behavior of the spin system is a purely quantum effect, stemming from the symmetry of the ground state, the exact properties of the exchange Hamiltonian, and the known exact value of the contribution of the magnetic field to the spin Hamiltonian.

For definiteness we will conduct the discussion in terms of a collinear antiferromagnet; the ideas can easily be extended to other axial spin structures.

To bring out the feature of interest here, we consider the behavior of an easy-axis antiferromagnet in a longitudinal magnetic field at absolute zero. We ignore other relativistic effects (in addition to the uniaxial anisotropy) which would disrupt the axial symmetry and/or lead to nonconservation of the longitudinal spin projection. All states of the system are then characterized by certain integer values of the projection of the spin moment onto the symmetry axis. For a nonzero projection of the spin of an excitation, according to the symmetry of the antiferromagnet (there is necessarily a symmetry element which sends the oppositely oriented sublattices into each other and which simultaneously changes the sign of the spin projection of the excitation), the magnetic branches are doubly degenerate in a zero field.

Each state of the system, both the ground state and each excited state, remain eigenstates for the Hamiltonian which incorporates the effect of the magnetic field. The contribution of this field to the exact microscopic Hamiltonian reduces to the single term

$$-2\mu\mathbf{H}\cdot\mathbf{S} \tag{1}$$

if we ignore other, weaker (in terms of the fine-structure constant) relativistic effects. Here μ is the spin magnetic moment of the free electron, and the operator **S** represents the total spin moment. The ground state of this system does not change at all (in particular, its zero magnetization does not change) until, in a finite field, the minimum energy of an excited state which has a different magnetization becomes smaller than the energy of the ground state, by virtue of the term in (1) (this is the point of the flipping of the sublattices).

A universal dependence, linear in the field, is characteristic of not only long-wave, low-frequency magnons (as follows from the theory of low-frequency spin dynamics[3]), but also magnons with an arbitrary quasimomentum and all other possible spin excitations (optical magnons and bound states of magnons) which can be characterized by a nonunit value of the spin projection onto the selected axis.

Clearly, these assertions are closely related to the form of the energy in (1). For example, if there were some contribution in addition to (1), say, one of the form

$$\alpha\Sigma_i(\mathbf{S}_i\cdot\mathbf{H})^2, \tag{2}$$

where the operator $\mathbf{S}_i$ represents the spin of the ith atom, and α is a constant, then the assertion that the longitudinal susceptibility is zero would no longer be correct. There would then be a finite longitudinal susceptibility even in a completely polarized ferromagnet, no matter how strange that may sound. Here we should recall that there is a well-known precedent, of an equally perplexing behavior of quantum systems: Langevin diamagnetism. In this case the magnetization of an atom arises in a state with a zero projection of the orbital angular momentum onto the magnetic field, because of a specific contribution of the magnetic field to the Hamiltonian (§113, Ref. 4).

We would like to call attention to two new types of axial order of a spin system. These are last ones permissible on the basis of symmetry considerations. Purely spin elements of exchange symmetry[3] of the known axial spin structures form the following groups:

(C_∞, U_2R) (C_∞ is the axial-symmetry axis, U_2 is a twofold axis perpendicular to C_∞, and R is the operation of time reversal), for collinear magnetic materials (ferro-, ferri-, and antiferromagnets), which are characterized by an average spin density of the type[3]

$$\langle \mathbf{S}(\mathbf{r}) \rangle = \mathbf{l}\varphi(\mathbf{r}). \tag{3}$$

(C_∞, U_2, R), for spin nematic liquid crystals with a symmetric spin–spin correlation function of the type[5]

$$\langle S_i(\mathbf{r}_1) S_j(\mathbf{r}_2) \rangle = \varphi(\mathbf{r}_1, \mathbf{r}_2)\left(n_i n_k - \frac{1}{3}\delta_{ik} \right). \tag{4}$$

(C_∞, R), for spin nematic liquid crystals with a spin–spin correlation function which has an antisymmetric part of the type[5]

$$\langle S_i(\mathbf{r}_1) S_j(\mathbf{r}_2) \rangle = \varphi(\mathbf{r}_1, \mathbf{r}_2)\mathbf{P}_k e_{ijk}. \tag{5}$$

We know (§98 in Ref. 4; the operation R in our case replaces the spatial inversion discussed there) that in addition to these three types of axial groups, there are two more: C_∞ and (C_∞, U_2):

The group C_∞ corresponds to a state in which the ordinary spin vector $\mathbf{l}$ in (3) exists, and the two-point spin correlation function in (5) has a nonzero antisymmetric part. The vectors $\mathbf{l}$ and $\mathbf{P}$ are collinear.

The order parameter corresponding to the symmetry (C_∞, U_2) can be found without difficulty by examining a spin order of the tensor type;[6] specifically this symmetry corresponds to a tensor magnetic material described by a three-point spin correlation function of the type

$$\langle S_i(\mathbf{r}_1) S_j(\mathbf{r}_2) S_k(\mathbf{r}_3) \rangle = \varphi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) n_i n_l e_{ljk}, \tag{6}$$

where the function φ is transformed by a single representation of the symmetry group of the crystal.

Because of the flipping of the sublattices in antiferromagnets, spin nematic liquid crystals, and tensor magnetic materials, the rigidity of a spin system which we are discussing here (and which leads to the possibility of an exact quantitative description of the behavior in a magnetic field of both the ground state and the spectrum of excitations, without the introduction of any parameters) can be observed only in weak fields. In a collinear ferrimagnet, on the other hand, in a spherically symmetric scalar magnetic material,[6] and in the ground state of an exchange system without any symmetry breaking (which is possible in principle), the spin rigidity should also be observed in fields on the order of the exchange fields, until the energy of an optical magnon falls to zero, leading to an instability.

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