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On the Energy Spectrum of Helium II

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It is shown that the spectrum of the bulk excitations in helium II, which ends at the Pitaevskii point, should be recovered at a certain critical point with the coordinates of about several roton energies and momenta in the form of the spectrum of vortex rings. As the momentum increases, the spectrum of surface capillary waves should be transformed to the spectrum of surface vortex half-rings.

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According to Pitaevskii [1], the Landau phonon–roton spectrum $E(p)$ of elementary excitations in helium II (see Fig. 1) has an end point. At much higher energies and momenta, Rayfield and Reif [2] found an additional branch of excitations, or vortex rings. Pitaevskii [3], as well as Berloff and Roberts [4], discussed the possible behavior of the spectrum of vortex rings at energies of about the phonon–roton spectrum. In this work, we show that the experimental data allow a more specific conclusion on the spectrum of vortex rings at low energies.

The vortex ring has the same $C_{\infty v}$ symmetry as the phonon and roton: the axis of the infinite order around the momentum direction and reflection plane parallel to the momentum direction. For this reason, the indicated branches of the spectrum cannot intersect. Note that Feynman and Cohen [5] represented the structure of the atomic motion of excitation near the roton minimum in the microscopic theory of the helium spectrum as a vortex ring with a radius of about an atomic size. In principle, these excitations could form a unified continuous branch of the spectrum. However, since the phonon–roton spectrum in superfluid helium has an end point, the spectrum of vortex rings should either begin at a certain finite momentum or have a finite energy at zero momentum.

To estimate the parameters of the minimum vortex ring, we use the macroscopic approximation. In this approximation, the spectrum of vortex rings has the form [3] (see also [7])

$$E = \frac{\sqrt{\rho\kappa^3 p}}{4\sqrt{\pi}}\ln\frac{p}{\pi\kappa\rho\xi^2}. \quad (1)$$

This spectrum directly follows from the known Thomson results for vortex rings in classical hydrodynamics: the momentum of the ring with radius R is $p = \pi\kappa\rho R^2$ and its energy is $E = (\rho\kappa^2/2)R\ln(R/\xi)$, where κ is the circulation, ρ is the density of the fluid, and ξ is the quantity of the order of an interatomic distance. For superfluid helium, $\kappa = \hbar/m$, where m is the mass of the ^{4}He atom and ξ is the quantity of about a coherence length.

Investigating the mobility of charges in helium, Rayfield and Reif [2] observed the dependence of the velocity $v = \partial E/\partial p$ on the energy E determined by spectrum (1) (see Fig. 2). At high energies (1–50 eV), the charges of both signs are attached to vortex rings and move together. The parameter ξ determined from the data presented in [2] (see Fig. 2) is $\xi = 0.7 \pm 0.1$ Å. At this ξ value, with a decrease in the momentum, the region of instability with respect to the decay into six

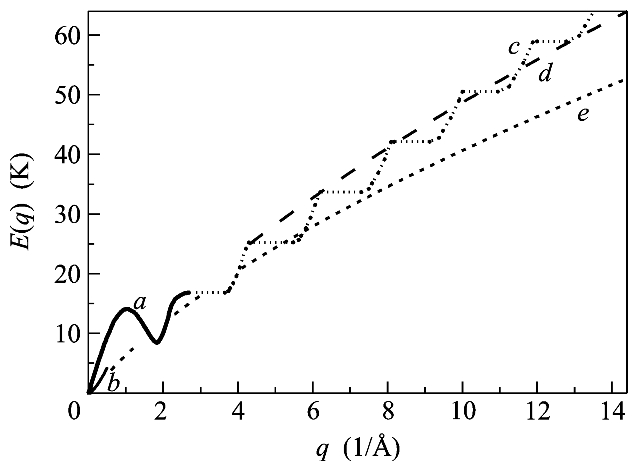

Fig. 1. Energy spectrum of helium II at zero pressure: line a is the phonon–roton spectrum (according to the data from [6]), line b is the spectrum of capillary waves on the free boundary, line c is the boundary of the stability of the possible additional excitations, line d is the vortex ring spectrum estimated by Eq. (1), and line e is the spectrum of the surface vortex half-rings estimated by Eq. (2). The energy E and wavenumber $q = p/\hbar$ are given in Kelvin and inverse angstrom, respectively.

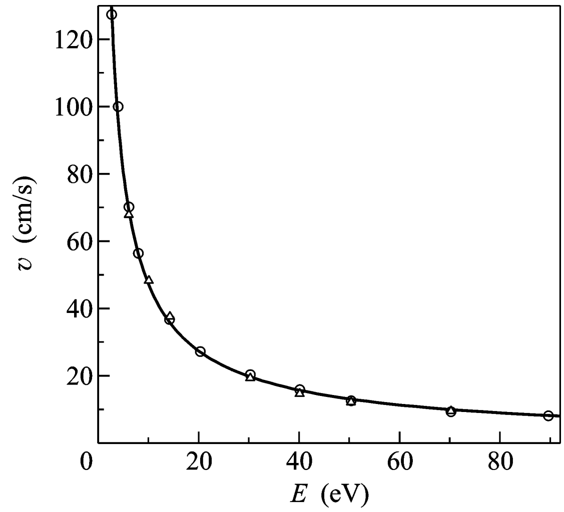

Fig. 2. Velocity of the vortex rings with attached (triangles) positive and (circles) negative charges [2]. The theoretical curve is plotted for $\xi = 0.7$ Å.

rotons appears in spectrum (1) at the point $q \approx 11.6$ Å^{-1} and $E \approx 54.5$ K (see Fig. 1). Following the procedure developed by Pitaevskii [1] for the determination of the features of the excitation spectra, one can easily verify that the damping of the excitation at zero temperature appears in this region. The diameter of the vortex ring at this point is approximately equal to 10.3 Å. Thus, the macroscopic approximation is roughly satisfied.

The instability region is bounded by basic spectrum line *a* and dotted line *c* presenting the minimum total energy of an arbitrary number of quasiparticles at a given total momentum. This line has alternating sections with decays into *n* identical quasiparticles and horizontal sections of decay into *n* rotons with zero velocity. Near the horizontal sections 3Δ (and 4Δ), there are small sections where decays occur into two (and three) quasiparticles with momenta slightly below the point of inflection (maximum of the group velocity) and one quasiparticle with a slightly higher momentum.

With a further decrease in the momentum, the stability region appears in the spectrum after the intersection of the horizontal section of the line of decay into six rotons with zero velocity. Then, the spectrum has the region of the possibility of decay into five rotons with a finite velocity, where the damping appears again, etc. down to the point ($q \approx 4, 3$ Å^{-1}, $E \approx 25$ K) of the intersection with the line of decay into two rotons with a finite velocity, where the spectrum of the vortex rings should begin according to the general result obtained by Pitaevskii [1]. The diameter of the minimum vortex ring is approximately equal to 6.3 Å Owing to an inaccuracy in the determination of the parameter ξ, the coordinates of the critical point of the beginning of the vortex-ring spectrum are estimated with an error of ~10%. Note that the point of the beginning of the damping region is determined with a lower accuracy, because the spectrum line in a wide energy range is close to the stability boundary.

As the pressure increases, a change in the parameters ρ, p_0, and Δ leads to a significant shift in the beginning point of the damping region in the spectrum of the vortex rings toward higher energies. In this case, the possibility that the vortex ring spectrum expands down to zero momentum is not excluded if its energy is sufficiently high.

Note that vortex half-rings whose spectrum in the macroscopic limit is determined directly from the spectrum of the bulk rings should propagate near the boundaries of helium II. Owing to the logarithmic divergence of the energy, a vortex approaching the boundary is oriented normally to it. Therefore, the moving vortex should be a half-ring with the momentum $p = \pi\kappa\rho R^2/2$ and energy

$$E_s = \frac{\sqrt{\rho\kappa^3 p}}{4\sqrt{2\pi}} \ln\frac{2p}{\pi\kappa\rho\xi^2}. \qquad (2)$$

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