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Magnetization of a two-dimensional electron gas

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The magnetic-field dependence of the oscillation amplitude of the magnetization of the two-dimensional electron gas is studied both analytically and numerically. A closed analytical expression for the envelope of the magnetization oscillations is obtained in the limit of sharp Landau levels.

The great success of the de Haas–van Alphen (dHvA) effect in three-dimensional conductors[1] is due to the existence of a well-developed theory, which provides a quantitative basis [the Lifshitz-Kosevich formula (LK) (Ref. 2)] for the determination of complicated Fermi surfaces of metals. The theory of the two-dimensional de Haas–van Alphen (2D dHvA) effect was started by Peierls.[3] In the zero-temperature limit he obtained that the magnetization in an ideal two-dimensional electron gas (2D EG) has a sharp, saw-tooth form and a constant, with field, amplitude. In this model the magnetization experiences jumps, at the end of each de Haas–van Alphen period, from a positive to a negative value and the susceptibility exhibits infinitely sharp negative spikes, respectively. For years this result was a standard textbook exercise. Recent progress in growing of two-dimensional conductors have attracted both the experimental[4–9] and the theoretical[10–18] attention to thermodynamic properties of two-dimensional electron gases.

The qualitative difference between the 3D and 2D dHvA effect follows from the fact that in the isotropic case the Fermi surface crosses a large number of Landau tubes, while in a 2D electron gas the Landau levels just below and just above the Fermi energy dominate the magnetic-field dependence of the chemical potential. This results in strong quantum oscillations of the chemical potential,[11,12,16] while in a three-dimensional case the quantum oscillations of the Fermi energy are negligibly small.[2] Vagner and co-workers[11,16] obtained an analytical expression for the magnetic-field dependence of the chemical potential and magnetization at finite temperatures in the limit of sharp Landau levels. Zawadski and Lassnig[12] have studied numerically the thermodynamics of 2D EG, assuming a Gaussian form of the Landau-level width. They have reported, also, strong quantum oscillations of the chemical potential, which define the oscillation form of the magnetization, if the electron density is field independent. Shoenberg[13] has performed an analytical calculation including the finite temperature and different forms of the Landau-level broadening. Several groups have applied the many-body techniques[14] and possible connections of the 2D dHvA with the quantum Hall effect were outlined in Refs. 11 and 15.

In spite of this growing activity in the theory of the two-dimensional dHvA, a simple analytical expression for the magnetic-field dependence of the magnetization of 2D EG, which could be used, like the Lifshitz-Kosevich formula, to analyze a wide spectrum of the experimental data is not yet available. In this Rapid Communication we derive an analytical expression for the envelope of the magnetization of the 2D electron gas at finite temperatures, in the limiting case of sharp Landau levels. While in the isotropic three-dimensional conductor the magnetization smoothly oscillates with field, in 2D EG one expects a sharp, saw-tooth magnetic-field dependence of magnetization. Such a different magnetic-field dependence of the magnetization dictates, obviously, a different mathematical approach to the problem. In the Lifshitz-Kosevich

theory the summation over the Landau levels is performed using the Poisson summation formula: a Fourier transform of a smooth function, operative when all but the first harmonics have negligible amplitudes. In the two-dimensional case the number of Fourier harmonics is expected to be relatively large. It is more convenient, therefore, to perform explicitly the summation over the Landau levels, since only two, adjacent to the Fermi energy, Landau levels are partially full.

Magnetization is defined by

$$M(B) = -V^{-1}\left(\frac{\partial F(B)}{\partial B}\right)_N \tag{1}$$

where $F(B) = \Omega(B) - \mu N$ is the Free energy, $\mu(B)$ is the chemical potential, and

$$\Omega = k_B T g(B) \sum_{n=0}^{\infty} \ln\{1+\exp[-(x_n+\alpha)]\} \tag{2}$$

is the thermodynamic potential (we neglect the spin contribution). Here $x_n \equiv (n\hbar\omega_c - \mu)/k_B T$, $\alpha = \hbar\omega_c/2k_B T$, n is the number of a Landau level, $g(B) = B/\phi_0$ is the degeneracy of a Landau level, $\omega_c = eB/m_c$ is the cyclotron frequency, and m_c is the effective (cyclotron) mass in the x-y plane. At low enough temperatures, $\alpha \gg 1$, the Landau levels far below the Fermi energy are completely occupied, while those far above the Fermi level are completely empty. If the number of particles is fixed, $N = \text{const}(B)$, the chemical potential is pinned to a Landau level during the entire dHvA period (the magnetization is linear with the field with a positive slope when the number of electrons is fixed) and jumps to a neighboring Landau level when a new dHvA period starts. The analytical expression for the magnetic-field dependence of the chemical potential at finite temperature is presented in Refs. 11 and 16.

Let us denote (we follow the Ref. 11) by n_F the value of n in Eq. (2) corresponding to the highest occupied level. Now $(n_F + \frac{1}{2})\hbar\omega_c > \mu > (n_F - \frac{1}{2})\hbar\omega_c$. Assuming $\hbar\omega_c \gg k_B T$ (i.e., $\alpha \gg 1$) we may neglect the unity in the summand of Eq. (2) for all n's up to $n = n_F - 2$. To take into account correctly, however, the depinning of the chemical potential from the level $(n_F + \frac{1}{2})\hbar\omega_c$ and the pinning to the level $(n_F - \frac{1}{2})\hbar\omega_c$ we should retain the exact expression for both the $(n_F - 1)$th and the n_Fth terms. Using this approximation, we arrive at the following analytical expression for the magnetization of the two-dimensional electron gas:[11]

$$M(B) = \frac{2}{\phi_0}\left[n_F\mu(B) + \hbar\omega_c A\left(n_F + \frac{\sinh\alpha}{2\sinh(x_F)}\right) - \hbar\omega_c(n_F^2+1) + k_B T \ln[2\cosh(x_F) + 2\cosh\alpha]\right], \tag{3}$$

where

$$A = \frac{\sinh x_F}{\cosh x_F + \cosh\alpha} = n_F - \frac{N}{Vg}$$

and

$$x_F = \frac{n_F\hbar\omega_c - \mu}{k_B T}.$$

This expression is a limiting case[19] of Eq. (28) in Ref. 13. Numerical calculation of the magnetization, based on Eq. (3), is presented in Fig. 1.

In the three-dimensional dHvA case the magnetization is well described by the LK formula,[1,2] and the amplitude of the oscillations is proportional to: $(\pi^2/\alpha)/\sinh(\pi^2/\alpha)$. This functional form follows from the Poisson summation

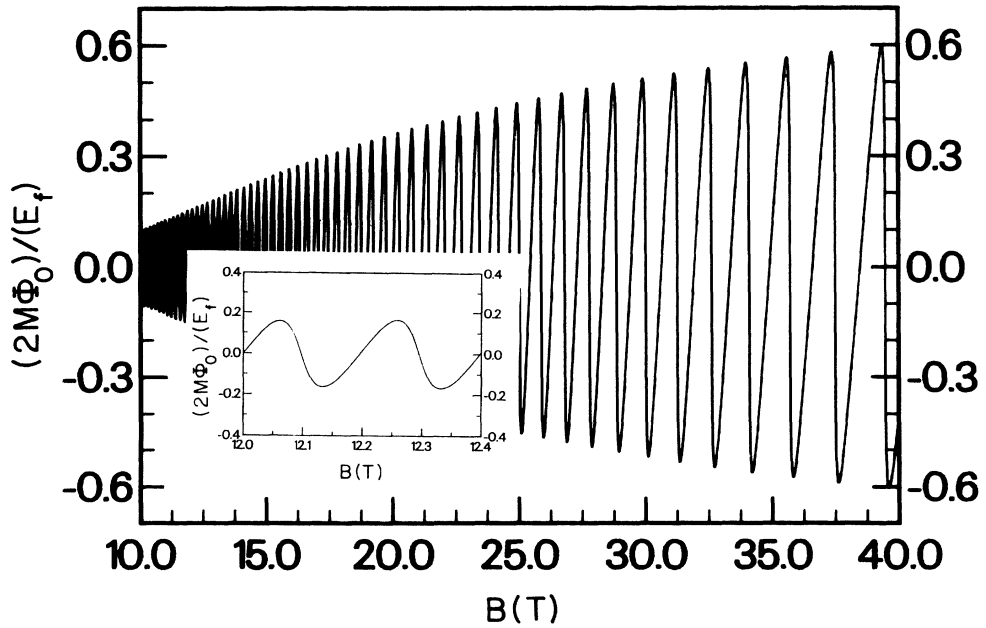

FIG. 1. Numerical calculation of the magnetization, based on Eq. (3). The parameters are as follows: $m_c = 0.3m_0$, where m_0 is the free-electron mass, $E_F = 0.3$ eV, and $T = 10$ K.

formula, used in LK theory to change the summation over the Landau levels into the integrals, when the Fermi surface crosses many Landau levels. However, in the 2D electron gas under sufficiently strong magnetic fields, when the energy gap (the Landau gap) $\hbar\omega_c$ is larger than the temperature smearing of the distribution function, the Poisson summation formula is not operative, since only two terms in the sum over n are contributing to the oscillation amplitude: these are the Landau levels just above and just under the Fermi energy.

Let us now define the functional form of the envelope of the magnetization in 2D EG at high magnetic fields and low temperatures, where $B_{\rm extr} \simeq B^*$. Here $B_{\rm extr}$ are the field values corresponding to the maxima (minima) of the magnetization, and B^* is the magnetic field when the chemical potential is exactly between the two Landau levels. In this case $|A|e^{\alpha} \gg 1$, and the magnetization Eq. (3) takes the form

$$M \simeq \pm \frac{2E_F}{\phi_0}\left[\frac{B}{2B^*}+\frac{1}{2\alpha^*}\ln\left[n_F\left|1-\frac{B^*}{B}\right|\right] - \left|\frac{B}{B^*}-1\right|\left[\left(n_F \pm \frac{1}{2}\right)\right]\right]. \quad (4)$$

Here E_F is the Fermi energy and $B_{\rm extr}$ can be found by equating to zero the derivative

$$\left.\frac{\partial M}{\partial B}\right|_{B_{\rm extr}} = 0:$$

$$B_{\rm extr} = B^*\left(1 \mp \frac{1}{2\alpha^*(n_F \pm 1)}\right). \quad (5)$$

Keeping in mind that $1/2\alpha^*(n_F \pm 1) \ll 1$, we substitute $B_{\rm extr}$, defined by Eq. (5), into Eq. (4) and obtain the analytical behavior of the magnetization amplitude:

$$M_{\rm extr} \simeq \pm \frac{E_F}{\phi_0}\left(1-\frac{1}{\alpha^*}\ln(2\alpha^*)-\frac{1}{\alpha^*}\right). \quad (6)$$

Figure 2 presents a comparison between the analytical expression Eq. (6) and the numerical calculation based on Eq. (3). In the low-field limit we use the LK formula, since at $\alpha < 1$ many Landau levels should be taken into account, and the Poisson summation formula can be effectively used.[13]

Let us discuss now the applicability of our main result, Eq. (6), in realistic experimental conditions. Our theory is valid in the limiting case of sharp Landau levels: $\hbar\omega_c > \Gamma, T, \Delta_z$, where Γ is the Landau-level width and Δ_z is the miniband width. We assume also that the electron density is field independent and zero density of state between the Landau levels. We consider here the experimental situation in the GaAs/Al_xGa_{1-x}As heterojunctions and in the GIC's—the graphite intercalated compounds.

The first dHvA measurements in GaAs/Al_xGa_{1-x}As superlattices and heterojunctions[4] have shown a relatively smooth magnetization, while a sharp, saw-tooth form of the magnetization should be expected: The inter-Landau-level spacing, at 5 T was $\hbar\omega_c \simeq 9$ meV and the Landau-level width was estimated, from the mobility measurements, to be $\Gamma \leq 2$ meV. These and several additional thermodynamic experiments[5] have raised the possibility of a substantial density of states between Landau levels. In Ref. 16 it was shown analytically that any finite density of states within the Landau gap removes the exponential sharpness of the negative magnetization slope in 2D EG. In Ref. 17 a statistical model for a spatially inhomogeneous two-dimensional electron gas was introduced, which yields an effective background density of states between the Landau levels. Unfortunately, the highly instructive and elegant experiments of the Bell Laboratories group[4] are still the only available source for the dHvA data in superlattices and single heterojunctions. We expect that the much sharper magnetization oscillations in samples with better mobilities could be analyzed using our Eq. (6).

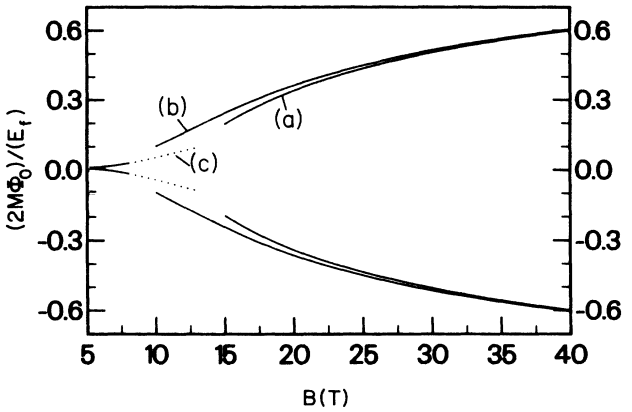

FIG. 2. (a) The high-field part of the envelope of magnetization, Eq. (3). (b) The analytical expression, Eq. (6). (c) The low-field region, as approximated by the LK formula.

Another family of the two-dimensional electron gases is presented by organic conductors[6] and by GIC's: the graphite intercalated compounds,[7-9] which are examples of a superlattice with a two-dimensional electron gas. The dHvA in a superlattice was studied theoretically in Ref. 10. Their obvious advantage for the dHvA measurements is a very high (almost metallic) electronic density. Their band structure and chemical composition complicates, however, the analyzing of the experimental data. We will discuss here recent experiments on As_5-GIC and Br_2-GIC.

Stage I: As_5-GIC. Joss and co-workers[7] have reported dHvA measurements on stage-I As_5-GIC in magnetic fields up to 22 T. They have found a cylindrical Fermi surface. The Landau-level separation at 20 T is found to be larger than their broadening. The oscillations are, however, rather smooth with higher harmonics. These data indicate that in As_5-GIC the equilibrium between the intercalant and the graphite layers is field dependent at high magnetic fields and the intercalant may serve as a flexible reservoir of carriers.[18] This will result in a triangular form of the magnetization oscillations, smoothed by temperature and imperfections. In this model only the odd harmonics in the Fourier spectrum should appear at high fields. Our theory, based on the assumption N

=const, is not applicable to this system, where neither N =const nor μ =const during a single dHvA period.

Stage II: Br_2-GIC. Markiewicz and co-workers[8] have reported sharp negative spikes in the susceptibility of a stage-II Br_2-GIC. Their conclusion is that this material presents a unique system with true energy gaps between Landau levels. This indicates that the magnetization measurements, if performed on stage-II Br_2-GIC, will show a sharp, saw-tooth, magnetic-field dependence, and could be effectively analyzed in the framework of our theory, i.e., using Eq. (6) for the magnetization envelope.

In conclusion, we have obtained an analytical expression for the magnetization oscillations envelope in the limiting case of sharp Landau levels and made a detailed comparison with the numerical calculation. Unlike in the LK theory we have not used the Poisson summation formula at low enough temperatures $\hbar\omega_c \gg k_B T$. In the high-field limit the agreement is rather good. In the low-field limit the Landau-level broadening cannot be neglected, and the Shoenberg theory[13] can be operative. We have analyzed the existing dHvA data in heterojunctions and GIC's and have outlined the conditions when our theory can be effectively used.

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