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On the Penetration Depth of a Strong Field into Superconductors

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The field dependence of the magnetic penetration depth over the entire range of stability and metastability of the Meissner state was determined within the framework of the Ginzburg–Landau theory. A simple interpolation formula is suggested. © *2003 MAIK "Nauka/Interperiodica".*

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Ginzburg and Landau [1] calculated the low-field correction to the magnetic penetration depth for an arbitrary value of the parameter κ:

$$\delta(H) = \delta(0)\left(1+\frac{\kappa(\kappa+2\sqrt{2})}{8(\kappa+\sqrt{2})^2}\frac{H^2}{H_c^2}\right). \tag{1}$$

In the limiting cases of large and small κ values, Eq. (1) is valid over the entire domain of existence of the equilibrium Meissner state (up to the field H_c in type-I superconductors and H_{c1} in type-II superconductors).

In type-I superconductors, the ordinary metal–superconductor transition in an external magnetic field is a phase transition of the first order. It can be accompanied by superheating or supercooling. In type-II superconductors, the process of superconductivity destruction involves an intermediate stage, during which the superconductor is in a mixed state. The transition from the normal to the mixed state in the field H_{c2} is a classical phase transition of the second order. The occurrence of the mixed state in a superconductor exposed to an external field H_{c1}, as well as its reconstruction in a varying magnetic field, should be accompanied by superheating or supercooling even in the absence of pinning. This is caused by the Bean–Livingston energy barrier to the vortex penetration into superconductors [2]. In the limit of large values of the Ginzburg–Landau parameter, the barrier to vortex penetration disappears in a field $H_m = H_c$ (see [3], §34) that considerably exceeds the field H_{c1}, in which the transition to the mixed state occurs.

A phase diagram (κ, H) for superconductors near the transition temperature, where the Ginzburg–Landau theory is valid, is shown in Fig. 1. The highest possible superheating field $H_m(\kappa)$ for the Meissner state was determined by Ginzburg [4], who solved numerically the Ginzburg–Landau equations.

In the limit of large κ values, Ginzburg obtained the analytical solution to the problem of magnetic field decay in the superconductor bulk:

$$B = 2H_c\sinh\left(\frac{x}{\delta}+C\right)\cosh^{-2}\left(\frac{x}{\delta}+C\right), \tag{2}$$

where the constant C is a function of the external field and determined by the following equation:

$$\cosh(C) = \frac{H_c}{H}\left(\sqrt{1+\frac{H}{H_c}}+\sqrt{1-\frac{H}{H_c}}\right). \tag{3}$$

The Ginzburg's solution determines the dependence of magnetic penetration depth

$$\delta(H) = \frac{1}{H}\int_0^\infty B\,dx$$

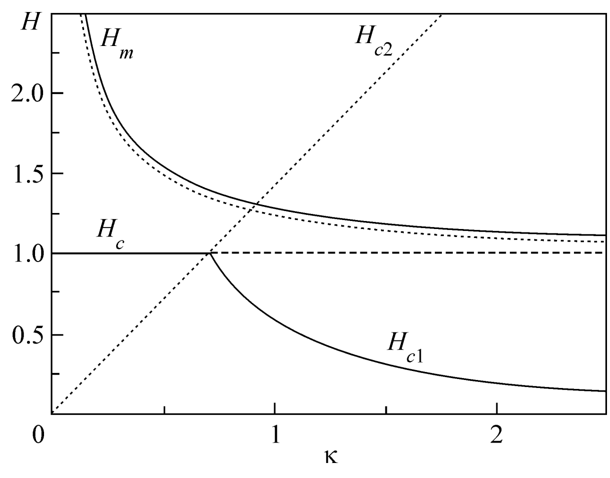

Fig. 1. Ginzburg–Landau phase diagram for superconductors. Solid curves (except H_c) for the maximal superheating field H_m and the field H_{c1} (equal to the ratio between the one-quantum vortex energy and the magnetic flux quantum) were obtained by numerical computation. The dotted curve is the function $H_{GL}(\kappa)$ given by Eq. (7).

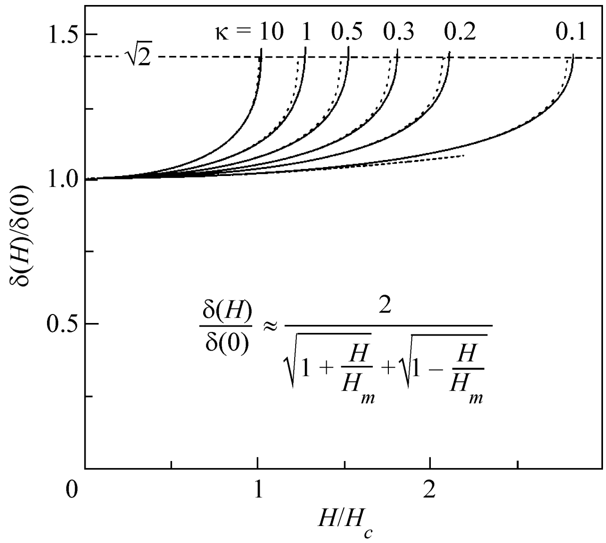

Fig. 2. Field dependence of the magnetic penetration depth for various values of the parameter κ. Solid lines are the results of numerical solution to the Ginzburg–Landau equations. Dotted lines are interpolation functions (4). The dashed line is the Ginzburg–Landau quadratic low-field approximation.

on a magnetic field over the entire range $0 < H < H_m = H_c$, where the Meissner state can be observed at large values of κ:

$$\delta(H) = 2\delta(0)\left(\sqrt{1+\frac{H}{H_m}}+\sqrt{1-\frac{H}{H_m}}\right)^{-1}. \quad (4)$$

In the limit of small κ values, the Ginzburg–Landau equation for the Meissner state was solved by Galaĭko [5]. For small κ, the presence of magnetic field at distances greater than the penetration depth can be neglected. Then,

$$\psi = \tanh\frac{\kappa(x-a)}{\sqrt{2}}. \quad (5)$$

The parameter a is determined by matching solution (5)
1 and the solution near the boundary of fieldpenetration region.

In the approximation considered, the field penetration is described by a simple exponential function $A = H|\psi_0|^{-1}\exp(-|\psi_0|x)$. It can easily be shown, using the solution obtained in [5], that in this limit the dependence of magnetic penetration depth $\delta(H) = \psi_0^{-1}$ on the external magnetic field is also given by Eq. (4) with the superheating field H_m equal to

$$H_m = \frac{H_c}{\sqrt{\sqrt{2}\kappa}}. \quad (6)$$

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Thus, quite unexpectedly, the field dependences for the small and large values of the parameter κ were found to coincide. The numerical solution to the Ginzburg–Landau equations for an arbitrary value of κ showed that the field dependence of the penetration depth over the entire field range deviated only slightly (Fig. 2) from the interpolation function (4), where the critical superheating field for an arbitrary κ is taken to be

$$H_m = H_{GL} = H_c\frac{\kappa+\sqrt{2}}{\sqrt{\kappa^2+2\sqrt{2}\kappa}}, \quad (7)$$

as follows from the low-field limit (1).

As the maximum superheating field is approached, both Eq. (4) and the numerical solution to the Ginzburg–Landau equations (Fig. 2) exhibit a root singularity. Such a singularity is usually indicative of a loss of stability of a metastable state against the uniform disturbances (see [6], part 3).

The close similarity between the results of numerical computation of the function $H_m(\kappa)$ and the curve $H_{GL}(\kappa)$ (Fig. 1), as well as the gact that the maximal 2
increase in the penetration depth is close to $\sqrt{2}$ (Fig. 2), show that the accuracy of the suggested interpolation formula is rather high. Thus, the superconductor parameters H_c and κ can reliably be determined from the measured field dependence of the penetration depth in bulk superconductors only in the combination entering the equation for H_{GL}. To separately determine these parameters with an accuracy of 10^{-1}, the accuracy of measurement of the penetration depth should be 10^{-3} or better.

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