ISSN 0021-3640, JETP Letters, 2007, Vol. 86, No. 11, pp. 730–731. © Pleiades Publishing, Ltd., 2007.
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On Twinning in Smectic Crystals

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Received October 6, 2007

It is shown that mechanical twinning in smectic crystals is possible. The structure of the boundary of twins for a small disorientation of crystallites is determined. The periodic twin structure, which should appear at the tension of the smectic layer, is proposed.

PACS numbers: 61.30.-v
DOI: 10.1134/S0021364007230105

The energy of small deformations of a smectic crystal is given by the expression [1]

$$\mathcal{E} = \int \frac{A}{2}\left\{\left(\partial_z u - \frac{(\partial_\alpha u)^2}{2}\right)^2 + \lambda^2(\Delta_\perp u)^2\right\} dV, \qquad (1)$$

where u is the displacement of the layers along the z axis (in the initial homogeneous undeformed state of the smectic crystal, layers lie in the xy plane), A is the elastic modulus, λ is the length parameter, ∂_α is the gradient vector in the xy plane, and $\Delta_\perp = \partial_\alpha^2$. According to Eq. (1), the undeformed state turned by small angle $\theta \ll 1$ in the xz plane (in this case, $\partial_x u = \theta$) corresponds to the derivative $\partial_z u = \theta^2/2$.

Let us consider the boundary between the states $\partial_x u = \pm\theta$ ($x \longrightarrow \pm\infty$) that lies in the yz plane. The quantity $\partial_z u$ is unchanged inside the boundary. The variation equilibrium equation in the problem under consideration reduces to the form

$$\lambda^2 f''' + \frac{\theta^2}{2} f' - \frac{3}{2} f^2 f' = 0, \qquad (2)$$

where $f = \partial_x u$. The solution of this equation has the form $f = \theta \tanh(qx/2\lambda)$. The energy of the unit area of this boundary is given by the formula

$$\sigma = \frac{2}{3} A \lambda \theta^3. \qquad (3)$$

The twin structure of smectic crystals must be observed under the conditions of Helfrich instability at strains noticeably larger than the critical value (see the problem in [1, Sect. 45]: the smectic layer of thickness L bounded by solid walls parallel to the smectic layer is extended along the z axis). At very small tensions $\delta L > \delta L_c = 2\pi\lambda$, i.e., when δL is about the smectic period, the homogeneous state becomes unstable with respect to the appearance of a periodic structure in the xy plane with wavenumber $k_c = \sqrt{\pi/\lambda L}$. At a much larger strain $\delta L \gg \delta L_c$, a twin structure as that schematically shown in the figure should appear. The parameters of this structure are determined by minimizing the total energy of twin boundaries ($\theta = \varepsilon$ at the vertical boundaries and $\theta = \varepsilon/2$ at the boundaries of the triangular regions). According to geometric consideration, the angle at the vertex of a triangle is equal to ε, the height H of the triangles is related to the structure period d as $\tan(\varepsilon/2) = d/2H$, and the quantity δL is related to the parameter ε as $\delta L = (L - H)(\cos^{-1}\varepsilon - 1)$. The energy density of the structure proposed above is given by the expression

$$\frac{1}{Ld}\left\{2\frac{L-H}{\cos\varepsilon}\sigma(\varepsilon) + 4\frac{H}{\cos(\varepsilon/2)}\sigma(\varepsilon/2)\right\}. \qquad (4)$$

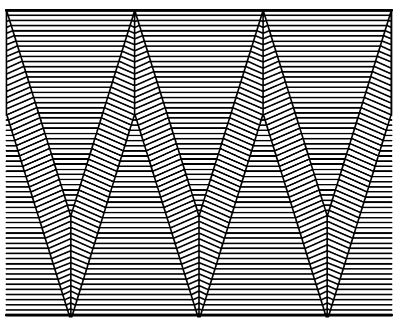

Figure.

In view of the indicated geometric relationships, energy (4) is a function of one parameter ε at a given tension $\delta L/L$. When $\delta L \ll L$, angle ε is small. In this case, with the use of result (3), the minimum of energy (4) is found to correspond to $\varepsilon = \sqrt{6\delta L/L}$. In this case, $H = 2L/3$ and $d = 2\sqrt{2L\delta L/3}$.

I am grateful to E.I. Kats for stimulating discussions.

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Translated by R. Tyapaev

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