

# “Anderson transition” in superconducting superlattices

I. M. Suslov

*P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, 117924, Moscow*

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Superconducting superlattices consisting of alternating layers of two materials, with thicknesses  $d_0$  and  $d_1$  which are small in comparison with the coherence length, are analyzed. An analog of the de Gennes and Cooper equations is derived for the case of clean superlattices consisting of metals differing in effective mass and Fermi momentum, for an arbitrary transmission of the interfaces. Under certain conditions, there is a localization of the order parameter in the layers of one of the materials, and an “Anderson transition” can occur from a localized state of a delocalized state as  $d_0$  or  $d_1$  is varied.

## 1. INTRODUCTION

Some recent papers<sup>1,2</sup> analyzing the mechanism for the superconductivity of twin planes and interpreting the “superconductivity explosion” of 1987 have taken up the possibility of a localization of the order parameter in small regions of coordinate space and the possible occurrence of an “Anderson transition” from a localized state to a delocalized one upon a variation in the parameters of the system. Attempts to directly observe these effects experimentally, however, run into the difficulty that in practice it is not a simple matter to arrange the necessary variations in the parameters in the situations discussed in Refs. 1 and 2. As we will show below, these effects can be easily observed in superconducting superlattices.

Interest in experimental<sup>3,4</sup> and theoretical<sup>5–7</sup> research on superconducting superlattices has recently been heightened by the development of new technological deposition methods (molecular beam epitaxy, among others) and especially by the discovery of the high  $T_c$  oxide superconductors,<sup>8</sup> since the layered structure of the latter materials makes superlattices suitable models for studying the properties of those materials.

We consider a superlattice consisting of alternating layers of materials 0 and 1, with thicknesses  $d_0$  and  $d_1 \equiv d$  and with an overall period  $L = d_0 + d_1$ . The constant  $V$  of the four-fermion interaction of the BCS theory and the density of electron states at the Fermi level,  $N$ , take on the values  $V_0, N_0$  and  $V_1, N_1$ , respectively, in the layers. The Debye energies  $\omega_D$  of materials 0 and 1 are assumed to be identical. This approximation is justified in a weak-coupling theory because of the comparatively weak  $\omega_D$  dependence of the transition temperature  $T_c$  of the superlattice.

If  $d_0$  and  $d_1$  are large in comparison with the coherence length  $\xi_0$ , the superlattice can be described by the Ginzburg-Landau theory with suitable boundary conditions.<sup>9</sup> We are instead interested in the opposite case,  $d_0, d_1 \ll \xi_0$  (but  $d_0, d_1 \gg a$ , where  $a$  is the interatomic distance). In this case,  $T_c$  of the superlattice is given by the BCS formula  $T_c \sim \omega_D \exp(-1/\lambda_{\text{eff}})$ , where  $\lambda_{\text{eff}}$  is given in the case  $N_0 = N_1 \equiv N$  by the Cooper formula<sup>10</sup>

$$\lambda_{\text{eff}} = \frac{\lambda_0 d_0 + \lambda_1 d_1}{d_0 + d_1}; \quad \lambda_0 = V_0 N; \quad \lambda_1 = V_1 N. \quad (1)$$

In other words, it is found by taking a simple spatial average of the quantity  $\lambda = VN$ . This result remains in force for an arbitrary coordinate dependence of  $V$  (Ref. 11). For extremely dirty superconductors, expression (1) was generalized by de Gennes<sup>12</sup> to the case<sup>11</sup>  $N_0 \neq N_1$ :

$$\lambda_{\text{eff}} = \frac{V_0 N_0^2 d_0 + V_1 N_1^2 d_1}{N_0 d_0 + N_1 d_1}. \quad (2)$$

In the present paper we are interested in clean superconductors with  $N_0 \neq N_1$ , which is a case not covered by Eqs. (1) and (2). As in Eqs. (1) and (2), the quantity  $\lambda_{\text{eff}}$  turns out to depend

on only the ratio  $d_0/d_1$ , and it lies between  $\lambda_0$  and  $l_1$ . The nature of the averaging of  $\lambda$  depends strongly on the penetrability of the interfaces between materials 0 and 1. The localization of the order parameter which we mentioned above is seen in the circumstance that under certain conditions  $\lambda_{\text{eff}}$  does not tend toward  $\lambda_0$  as  $d_1 \rightarrow 0$  (under the conditions for the applicability of these equations,  $d_1 \gg a$ ). The Anderson transition is seen as a slope change on the plot of  $T_c$  versus  $d_1$  (under the condition  $L = \text{const}$ ). It stems from an abrupt restructuring of the order parameter in a small interval of  $d_1$  values. These effects obviously do not occur under conditions such that Eqs. (1) and (2) are applicable.

## 2. PROPERTIES OF THE SUPERCONDUCTING CORE

We adopt the direction of the spatial variation of the superlattice as the  $z$  axis, so the parameters  $V$  and  $N$  are functions of  $z$ . In calculating  $T_c$  of the superlattice we ignore surface effects; i.e., we ignore the detailed behavior of physical quantities over distances  $\sim a$  from the interfaces. At the same accuracy level we can discuss a two-layer sandwich in place of a superlattice. We assume that the region  $0 < z < d$  is filled with material 1, and the region  $d < z < L$  with material 0.

Under the condition  $L \ll v_F/\omega_D$  ( $v_F$  is the Fermi velocity) the kernel  $K(z, z')$  in the Gor'kov equation

$$\Delta(z) = V(z) \int K(z, z') \Delta(z') dz' \quad (3)$$

can be regarded as piecewise-constant<sup>2)</sup> [like the function  $V(x)$ ]:

$$K(z, z') = \begin{cases} K_{11}, & 0 < z, z' < d, \\ K_{00}, & d < z, z' < L, \\ K_{01}, & (z-d)(z'-d) < 0, \end{cases} \quad V(z) = \begin{cases} V_1, & 0 < z < d, \\ V_0, & d < z < L. \end{cases} \quad (4)$$

We can then seek a solution of (3) in the piecewise-constant form

$$\Delta(z) = \begin{cases} \Delta_1, & 0 < z < d, \\ \Delta_0, & d < z < L. \end{cases} \quad (5)$$

The temperature dependence of  $K_{ij}$  turns out to be logarithmic (§4):

$$K_{ij} = L_{ij} \ln \frac{1,14 \omega_D}{T}. \quad (6)$$

The equation for  $T_c$  is

$$1 - (V_0 L_{00} d_0 + V_1 L_{11} d_1) \ln \frac{1,14 \omega_D}{T} + V_0 V_1 d_0 d_1 (L_{00} L_{11} - L_{01}^2) \ln^2 \frac{1,14 \omega_D}{T} = 0. \quad (7)$$

From the sum rules<sup>12</sup> for the kernel  $K(z, z')$

$$\int K(z, z') dz' = N(z) \ln \frac{1,14 \omega_D}{T} \quad (8)$$

we find two equations relating  $L_{00}$ ,  $L_{11}$ , and  $L_{01}$ :

$$L_{00} d_0 + L_{01} d_1 = N_0; \quad L_{01} d_0 + L_{11} d_1 = N_1. \quad (9)$$

For dirty superconductors, the de Gennes diffusion equation<sup>12</sup> yields a third relation among the  $L_{ij}$ . It thus becomes possible to derive Eq. (2). For clean superconductors one of the quantities  $L_{ij}$  must be calculated from a microscopic model.

### 3. MICROSCOPIC MODEL

For materials 0 and 1 we adopt isotropic quadratic spectra of the general form

$$\varepsilon_0(\mathbf{k}) = U_0 + \frac{k^2}{2m} \equiv \frac{k^2 - k_0^2}{2m}; \quad \varepsilon_1(\mathbf{k}) = U_1 + \frac{k^2}{2m_1} \equiv \frac{k^2 - k_1^2}{2m_1}. \quad (10)$$

The energy is reckoned from the Fermi level, so  $K_0$  and  $K_1$  are the corresponding Fermi momenta. By virtue of the separation of variables in the Schrödinger equation, the one-particle wave functions are of the form  $\varphi(z)\exp(iK_{\parallel}r_1)$ , where  $r_{\parallel} = (x, y)$  and  $K_{\parallel} = (K_x, K_y)$ . If the unit cells of materials 0 and 1 are identical in volume, the boundary conditions at the interface (at the  $z = d$  plane for a sandwich) are<sup>16</sup>

$$\varphi(d + 0) = \varphi(d - 0); \quad \varphi'(d + 0) - \frac{m}{m_1} \varphi'(d - 0) = \kappa \varphi(d). \quad (11)$$

The parameter  $\kappa$  is a characteristic of the interface. It determines the transmission coefficient of the interface,  $D$ :

$$\begin{aligned} D &\sim k_F^2/\kappa^2 \quad \text{for } \kappa^2 \geq k_F^2; \\ D &\sim 1 \quad \text{for } \kappa^2 \lesssim k_F^2 \end{aligned} \quad (12)$$

In the estimates we assume  $k_0 \sim k_1 \sim k_F \sim a^{-1}$  and  $m \sim m_1$ . We are ignoring the dependence of  $\kappa$  on  $k_{\parallel}$ , as we are justified in doing near the band edge.

The one-particle eigenfunctions  $\varphi_{sk_{\parallel}}(z)$  and the eigenvalues  $\varepsilon_s(k_{\parallel})$  are classified in terms of the transverse quantum number  $s$  and the longitudinal quasimomentum  $k_{\parallel}$ . At a fixed  $k_{\parallel}$ , states  $s$  of the superlattice may belong to any of the following: a continuous spectrum, a quasisdiscrete spectrum, and a discrete spectrum. In general, one can distinguish four spectral regions (Fig. 1): region I, in which the states belong to the continuum and propagate throughout the superlattice; region II, in which the states are localized within the layers of material 1 and form a quasisdiscrete spectrum because of a quantum size effect (the overlap of the wave functions corresponding to neighboring layers of material 1 is ignored); region III, which contains states of a quasisdiscrete spectrum which are localized in the layers of material 0; and region IV, which contains surface states (Tamm states) which belong to a discrete spectrum and which are localized near the interfaces. For definiteness we assume  $k_1 > k_0$ .

For a two-layer sandwich with homogeneous boundary conditions at the points  $z = 0$  and  $z = L$ , the expressions for  $\varphi_{sk_{\parallel}}(z)$  and  $\varepsilon_x(k_{\parallel})$  in region I are

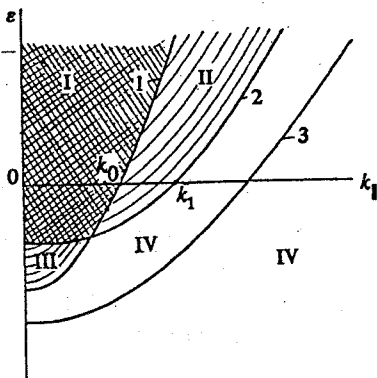


FIG. 1. Spectrum of a superlattice for  $m, m_1 > 0$ . Line 1 [ $\varepsilon = (k_{\parallel}^2 - k_0^2)/2m$ ] and line 2 [ $\varepsilon = (k_{\parallel}^2 - k_1^2)/2m$ ] break up the  $(\varepsilon, k_{\parallel})$  plane into four regions. Region I—continuous spectrum (the structure of this spectrum which is associated with the splitting into minibands is not shown); II, III—quasisdiscrete spectrum (consisting of a set of two-dimensional bands); IV—the region outside the spectra of materials 0 and 1, in which there are surface states (line 3).

$$\varphi_{sk_{\parallel}}(z) = \begin{cases} \frac{\sqrt{2} \sin k_s d_0}{\sqrt{d \sin^2 k_s d_0 + d_0 \sin^2 q_s d}} \sin q_s z, & 0 < z < d; \\ \frac{-\sqrt{2} \sin q_s d}{\sqrt{d \sin^2 k_s d_0 + d_0 \sin^2 q_s d}} \sin k_s(z - L), & d < z < L; \end{cases} \quad (13)$$

$$\varepsilon_s(k_{\parallel}) = (k_s^2 + k_1^2 - k_0^2)/2m = (q_s^2 + k_1^2 - k_1^2)/2m_1; \quad (14)$$

$$2k_s d_0 = 2\pi s + \pi + 2 \operatorname{arccot} \left( \frac{\kappa}{k_s} + \frac{m}{m_1} \frac{q_s}{k_s} \cot q_s d \right). \quad (15)$$

#### 4. CALCULATION OF $L_{01}$

The kernel  $K(z, z')$  is expressed in terms of  $\varphi_{sk_{\parallel}}(z)$  and  $\varepsilon_s(k_{\parallel})$ :

$$K(z, z') = T \sum_{|\omega| < \omega_D} \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \sum_{ss'} \frac{\varphi_{sk_{\parallel}}(z) \varphi_{sk_{\parallel}}^*(z') \varphi_{s'k_{\parallel}}(z) \varphi_{s'k_{\parallel}}^*(z')}{i\omega - \varepsilon_s(k_{\parallel})} \frac{1}{-i\omega - \varepsilon_{s'}(k_{\parallel})}. \quad (16)$$

As Kirzhnits and Maksimov have shown,<sup>11</sup> under the condition  $L \ll v_F/\omega$  we can discard terms with  $s \neq s'$  in (16). That this simplification is legitimate can be verified by noting that for  $s \neq s'$  we have  $|\varepsilon_s(k_{\parallel}) - \varepsilon_{s'}(k_{\parallel})| \gg \omega_D$  and by evaluating the terms in the sum over  $s$  and  $s'$ , ignoring the dependence of  $\varphi_{sk_{\parallel}}(z)$  on  $k_{\parallel}$ . Incorporating the latter dependence does not change the estimates of the terms with  $s = s'$ ; it can only reduce the terms with  $s \neq s'$ . As an example, let us calculate  $K_{01}$ . Assuming  $0 < z < d$  and  $d < z' < L$  (see endnote<sup>31</sup>) in (16), and assuming that  $z$  and  $z'$  are separated from interfaces by distances large in comparison with the interatomic distances, we see that only region I contributes. Substituting in (13)–(15), and noting that the vicinity of the points  $q_s = q_f$ ,  $k_s = k_f$ , where

$$q_f = \sqrt{k_1^2 - k_1^2}; \quad k_f = \sqrt{k_0^2 - k_1^2}, \quad (17)$$

contributes to the sum over  $s$ , we expand the slowly varying functions in  $q_s - q_f$  and  $k_s - k_f$ . Expressing  $\sin k_s d_0$  in terms of  $\cot q_s d$  and using Eq. (15), we find a sum over the wave vectors  $q_s$  which are the solutions of the equations  $W(q_s) = s$  with the following function  $W(q)$ :

$$2\pi W(q) = \pi + 2d_0 \left[ -k_f + \frac{m}{m_1} \frac{(q_f - q)q_f}{k_f} \right] + 2 \operatorname{arccot} \left[ \frac{\kappa}{k_f} + \frac{mq_f}{m_1 k_f} \cot qd \right]. \quad (18)$$

Using the transformation

$$\begin{aligned} \sum_{s=-\infty}^{\infty} F(q_s) &= \int_{-\infty}^{\infty} dq F(q) \sum_{s=-\infty}^{\infty} |W'(q_s)| \delta(W(q) - s) \\ &= \int_{-\infty}^{\infty} dq F(q) |W'(q)| \sum_{s=-\infty}^{\infty} e^{i2\pi s W(q)}, \end{aligned} \quad (19)$$

we find ( $v_f = q_f/m_1$ )

$$K_{01}(z, z') = T \sum_{\omega} \int_{|k_{\parallel}| < k_0} \frac{d^2 k_{\perp}}{(2\pi)^2} \int dq \frac{q_f}{\pi k_f} \frac{m}{m_1} \times \frac{4 \sin^2 qz \sin^2 \left[ \left( k_f + \frac{m}{m_1} \frac{(q - q_f) q_f}{k_f} \right) (z' - L) \right]}{\left[ d + d_0 f(qd) \right] \left[ \omega^2 + v_f^2 (q - q_f)^2 \right]} \sum_{s = -\infty}^{\infty} e^{i2\pi s W(q)}, \quad (20)$$

where

$$f(qd) = \sin^2 qd + \left[ \frac{\kappa}{k_f} \sin qd + \frac{m}{m_1} \frac{q_f}{k_f} \cos qd \right]^2. \quad (21)$$

We expand the periodic functions of  $qd$  in Fourier series, introduce a shift  $q \rightarrow q + q_f$ , and discard the terms which contain rapidly oscillating factors  $e^{i2sqd}$ ,  $e^{isk_f d}$  with  $s \neq 0$ ,  $e^{i2q_f}$ , etc., which vanish upon the integration over  $k_{\parallel}$ . As a result, we are left with only the term with  $s = 0$  in (20), and the dependence on  $z, z'$  disappears:

$$K_{01} = \ln \frac{1,14 \omega_D}{T} \int_{|k_{\parallel}| < k_0} \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{|m|}{\pi^2 k_f} \int_0^{\pi} \frac{dy}{d + d_0 f(y)}. \quad (22)$$

Evaluating the other integrals, we find the following expression for  $L_{01}$ :

$$L_{01} = \frac{|m_1|}{2\pi^2 d_0} \times \frac{\sqrt{k_1^2 + \frac{d}{d_0} \left[ k_1^2 + \frac{m_1^2}{m^2} (k_0^2 + \kappa^2) \right]} + \frac{d^2}{d_0^2} \frac{m_1^2}{m^2} k_0^2} - \sqrt{(k_1^2 - k_0^2) + \frac{d}{d_0} (k_1^2 - k_0^2 + \frac{m_1^2}{m^2} \kappa^2)}}{1 + \frac{d}{d_0} \left( 1 + \frac{m_1^2}{m^2} \right) + \frac{d^2}{d_0^2} \frac{m_1^2}{m^2}}. \quad (23)$$

Expression (23) along with (7) and (9) gives us a complete solution of our problem of calculating  $T_c$  of the superlattice.

## 5. ANALYSIS OF THE EQUATIONS DERIVED HERE

It is easy to see from (7), (9), and (23) that  $T_c$  is given by the BCS formula  $T_c = 1.14 \omega_D \exp(-1/\lambda_{\text{eff}})$ , where  $\lambda_{\text{eff}}$  is a function of only the ratio  $d/d_0$ , not of  $d$  or  $d_0$  separately. With  $m = m_1$ ,  $k_0 = k_1$ , and  $\kappa = 0$ , the expression for  $\lambda_{\text{eff}}$  becomes Cooper's formula, (1).

With  $k_1 \sim k_0 \sim k_1 - k_0$  and  $m \sim m_1$ , the structure of the radicals in (23) distinguishes three regions in which simple asymptotic expressions can be derived: the regions  $d/d_0 \ll D$ ,  $D \ll d/d_0 \ll D^{-1}$ , and  $d/d_0 \gg D^{-1}$  (for  $D \sim 1$ , the second of these regions does not exist). Setting  $\lambda_0 = V_0 N_0$ , and  $\lambda_1 = V_1 N_1$ , and introducing

$$T^* = 1,14 \omega_D e^{-1/\lambda^*}; \quad \lambda^* = V_1 N^*, \quad (24)$$

where  $N^*$  is the density of states at the Fermi level corresponding to region II of the quasidiscrete spectrum,

$$N^* = \int_{k_{\parallel} > |k_{\perp}| > k_0} \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{1}{d} \sum_s \delta \left( \varepsilon - \frac{q_s^2 + k_{\parallel}^2 - k_1^2}{2m_1} \right) \Big|_{\varepsilon=0} = \frac{|m_1|}{2\pi^2} \sqrt{k_1^2 - k_0^2}, \quad (25)$$

we find, in the region  $d/d_0 \ll D$ ,

$$\frac{T_c - T_{c0}}{T_{c0}} = \frac{V_0(N_1 - N^*)(V_1 N_1 - V_0 N_0)}{(V_0 N_0)^2 (V_0 N_0 - V_1 N^*)} \frac{d}{L}, \quad T_{c0} > T^*; \quad (26)$$

$$T_c = T^* + O\left(T^* \frac{d}{DL}\right), \quad T_{c0} < T^*. \quad (27)$$

In the region  $D \ll d/d_0 \ll D^{-1}$  we find

$$\frac{T_c - T_{c0}}{T_{c0}} = -\frac{1}{2V_0 N_0} \sqrt{\frac{d}{d_0} \frac{k_0^2}{\kappa^2}}, \quad T_{c0} > T_{c1};$$

$$\frac{T_c - T_{c1}}{T_{c1}} = -\frac{V_1 N_0}{2(V_1 N_1)^2} \sqrt{\frac{d_0}{d} \frac{k_0^2}{\kappa^2}}, \quad T_{c0} < T_{c1}.$$

In the region  $d/d_0 \gg D^{-1}$  we find

$$\frac{T_c - T_{c1}}{T_{c1}} = \frac{V_1 N_0 (V_0 N_0 - V_1 N_1)}{(V_1 N_1)^3} \frac{d_0}{L}. \quad (28)$$

Figure 2 shows  $T_c$  versus  $d$  at a constant  $L$  for various relations among  $T_{c0}$ ,  $T_{c1}$ , and  $T^*$  (in all cases, the relation  $T^* < T_{c1}$  holds). Here are the most characteristic features of these curves: (a)

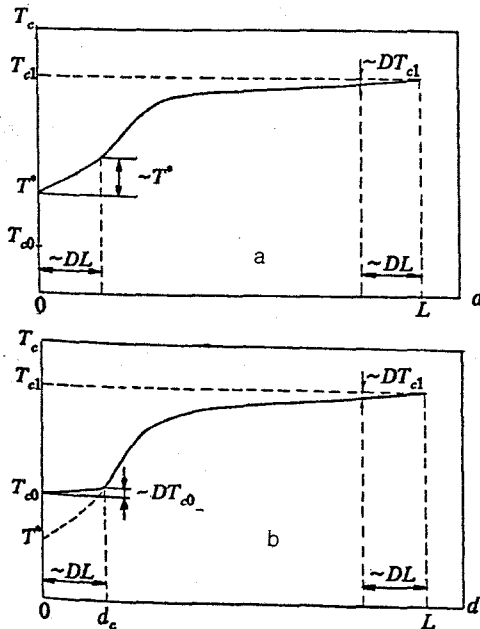


FIG. 2.  $T_c$  of the superlattice versus  $d$  at constant  $L$ . a— $T_{c1} > T^* > T_{c0}$ ; b— $T_{c1} > T_{c0} > T^*$ . The case  $T_{c0} > T_{c1}$  is similar to case (b) with  $T^* = 0$  and with an interchange of materials 0 and 1.

As  $D \rightarrow 0$  we find  $T_c = \max(T_{c1}, T_{c0})$ . This is just what we would expect to find on the basis of physical considerations, since the layers of materials 0 and 1 become independent in this case.

(b) The value of  $T_c$  lies between the transition temperatures  $T_{c0}$  and  $T_{c1}$  of the original materials, 0 and 1. (c) In the case  $T^* > T_{c0}$  (Fig. 2a),  $T_c$  does not tend toward  $T_{c0}$  as  $d \rightarrow 0$  (in the region of applicability,  $d \gg a$ ), because of a localization of the order parameter in the layers of material 1. A similar effect occurs for a planar defect in a superconductor,<sup>1,2</sup> in which case the effect stems from the presence of Tamm levels. In the case at hand, the role of the Tamm levels is played by states of the quasidiscrete spectrum which are localized in the layers of material 1. (d) In the case  $T_c > T^*$  (Fig. 2b), an "Anderson transition" occurs at the point  $d = d_c \sim D$ , from a regime of a localization of the order parameter in the layers of material 1 to a delocalized regime in which the order parameter propagates throughout the superlattice. The transition causes a slope change on the plot of  $T_c(d)$ . To study this transition we set

$$L_{01} \equiv \frac{N_1 - N_d}{d_0}; \quad T_d = 1,14 \omega_D e^{-1/V_1 N_d}, \quad (29)$$

express  $L_{00}$  and  $L_{11}$  in terms of  $L_{01}$  in accordance with (9), and substitute the results into (7). In the region  $d \ll d_0$  we can omit the terms  $\sim d/d_0$ ; we find  $T_c = \max(T_{c0}, T_d)$  (see endnote<sup>4</sup>). Since  $N_d$  changes from  $N^*$  to  $N_1$  with increasing  $d$ , the quantity  $T_d$  (see endnote<sup>5</sup>) changes from  $T^*$  to  $T_{c1}$ , and at a certain point  $d_c$  it intersects  $T_{c0}$ . In this approximation the curve of  $T_c(d)$  thus has a slope change at  $d = d_c$ . From the Gor'kov equation (3), with the kernel in (4), we easily find

$$\Delta_1 = \Delta_0 \frac{V_1 K_{01} d_0}{1 - V_1 K_{1d}} = \Delta_0 \frac{V_1 (N_1 - N_0) N_d \ln(1,14 \omega_D / T_d)}{1 - V_1 N_d \ln(1,14 \omega_D / T_c)}. \quad (30)$$

For  $d < d_c$  we have  $T_c = T_{c0}$  and  $\Delta_1 \sim \Delta_0$ . For  $d > d_c$ , the denominator vanishes, and we have  $\Delta_1/\Delta_0 = \infty$ . When the terms  $\sim d/d_0$  which we discarded are taken into account—these terms are on the order of  $D$  near the transition—the slope change on the  $T_c(d)$  curve is stretched out over a distance  $\delta d \sim D^{1/2} d_c$ , and the ratio  $\Delta_1/\Delta_0$  is on the order of  $D^{-1}$  in the region  $d > d_c$ . At small values of  $D$  (which can be arranged by sputtering a thin layer of an insulator between materials 0 and 1), the transition is sharp. If a magnetic dopant is introduced in the layers of material 0, the curve of  $T_c(d)$  changes only in the region  $d < d_c$ . This property might be utilized to detect the transition.

For  $T_{c0} > T_{c1}$ , a corresponding transition occurs in the region  $d_0 \sim DL$ . This case is similar to the preceding case with  $T^* = 0$ , and with an interchange of materials 0 and 1.

I wish to thank A. F. Andreev for a discussion of these results.

<sup>1</sup>Strictly speaking, the topics in Refs. 10 and 12 were two-layer sandwiches, rather than superlattices. However, since  $T_c$  of a film of thickness  $d \ll \xi_0$  is the same, within surface effects  $\sim a/d$ , as  $T_c$  of the bulk material,<sup>13-15</sup> it is easy to see that  $T_c$  of a superlattice will be the same as  $T_c$  of a sandwich within terms  $\sim a/d_0$  and  $a/d_1$ . The opposite assertion in Ref. 7 is incorrect; in particular, it contradicts the results of Refs. 6 and 11.

<sup>2</sup>The assertion that  $K(z, z')$  is piecewise-constant was made by de Gennes<sup>12</sup> for dirty alloys with  $L \ll \xi \sim \sqrt{\xi_0 l}$  ( $l$  is the mean free path), but the stronger condition  $L \ll \sqrt{l v_F / \omega_D}$  was formally used in Ref. 12. In clean superconductors, property (4) is valid under the condition  $L \ll v_F / \omega_D$  ( $\phi 4$ ). At  $L \gtrsim v_F / \omega_D$ , this property does not hold, as is clear from simply the properties of the kernel  $K_0(r)$  for a homogeneous superconductor [ $K_0(r) \propto r^{-1}$  for  $v_F / \omega_D \lesssim r \lesssim \xi_0$ ]. Nevertheless, the scale value  $v_F / \omega_D$  is not manifested at all in several cases, and the results are valid over the broader region  $L \lesssim \xi_0$  (cf. Ref. 6).

<sup>3</sup>The Russian pages from which this translation was prepared were afflicted with many small omissions, apparently caused by a mispositioning of the pages during copying (translator's note).

<sup>4</sup>Another omission on Russian page. It's possible that this should be  $T_c = \max(T_{c0}, T_d)$ ; the subscript "d" and the "}" are guesses (translator's note).

<sup>5</sup>Another omission; the subscript "d" is a guess (translator's note).

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