A possibility to observe the Berezinskii law

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The Berezinskii localization law $\sigma(\omega) \propto -i\omega$ for frequency-dependent conductivity was never questioned, but never observed experimentally. We discuss several possibilities for observation of this law and the experimental difficulties arising at this way.

The localization law $\sigma(\omega) \propto -i\omega$ for frequency-dependent conductivity was predicted by Berezinskii in 1973 [1] for one-dimensional disordered systems. Ac- cording to self-consistent theory by Vollhardt and Wölfle [2], this law is valid in the localization phase for systems of arbitrary dimension d. In the recent paper [3] of the present author the same law was established for systems of finite size L at the arbitrary extent of disorder. The latter is a consequence of the fact that a finite system is topologically zero-dimensional, and its effective dimensionality is less than lower critical one ($d_{c1} = 2$).

The Berezinskii law was never questioned in the theoretical community, and simultaneously it was never observed experimentally. The reason for it was clarified in the paper [3]: the Berezinskii law is valid in closed systems, while the most of actual systems are open. In open systems, replacement $-i\omega \to -i\omega + \gamma$ occurs (where γ is inelastic damping) and the law $\sigma(\omega) \propto -i\omega$ transforms into the usual metallic behavior.

A possibility of realization of closed systems became clear after observation of the persistent current in disordered systems (in the Aharonov–Bohm geometry) [4, 5, 6], predicted in the paper [7]. In fact, the persistent current is a consequence of the Berezinskii law, leading to dissipativeless conductance. Its observation is possible, when a size L of the disordered ring is small in comparison with the inelastic length L_{in}, depending on temperature T. The typical scales in the indicated experiments were $L \sim 1\mu m$, $T \sim 100mK$. If one accepts that $L_{in} \propto T^{-2}$, then a system is closed for $L \lesssim 10nm$ in the helium region ($T \sim 1K$).

Let discuss several experimental situations, where observation of the Berezinskii law is possible.

1. The first variant is the island film of a disordered metal lying on the dielectric substrate (Fig.1). We suppose for clearance that all islands are of the same size L, which increases monotonically in the course of the film deposition [1]. Then for $L \lesssim L_{in}$ the Berezinskii law is valid (Fig.1,a), while in the opposite case $L \gtrsim L_{in}$ the usual metallic conductivity takes place (Fig.1,b). A transition from one regime to another can be provided by the change of L or the temperature.

At first glance, the described experiment is simple. However, there is a bottleneck in it. It is clear from relation $\epsilon \sim i\sigma/\omega$, that the law $\sigma \propto -i\omega$ corresponds to the frequency-independent dielectric permeability ϵ, so a disordered system is an ordinary dielectric. The properties of the film in the Berezinskii law regime are the same as those of the dielectric substrate, hence the former gives a negligible contribution to conductivity in background of the latter. The width of the film is by 6–7 orders less than the width of substrate, but the corresponding smallness can be partially compensated by a large value of the film permittivity ϵ_1 in comparison with its substrate value ϵ_0. By the order of magnitude, $\epsilon_1 \sim \xi^2/a_0^2$ (where ξ is the localization length for wave functions, and a_0 is the atomic space) and saturates by a value L^2/a_0^2 for large ξ. If the metallic film is weakly disordered [2], then for $L \sim 10nm$ its permittivity ϵ_1 can

[1] In fact, there is a distribution of islands in size, which shifts in the large L region in the course of deposition.

[2] The films are weakly disordered in the case of "simple" metals (such as Mg, Al, Sn), which are well-described by the pseudopotential theory [8]; a small pseudopotential provides weak scattering even in the amorphous state. Contrary, the

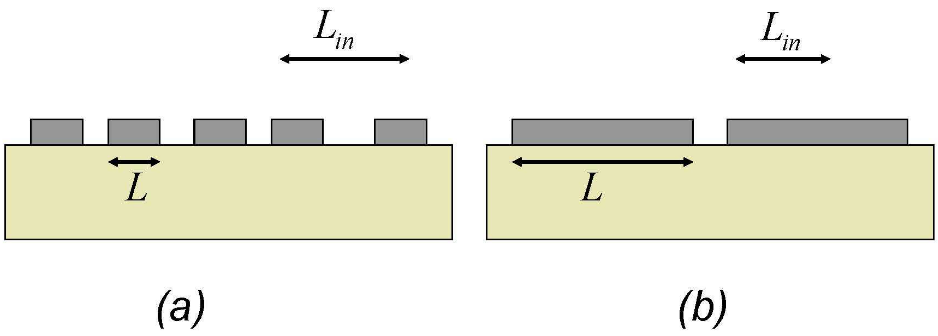

Figure 1: In the case of the island metallic film, the Berezinskii law is observable when the island size L is small in comparison with the inelastic length L_{in} (a), while in the opposite case the metallic behavior is valid (b).

exceed ϵ_0 by 3–4 orders.

The experimental procedure looks as follows. The experiment is carried out *in citu* and begins with a measurement of the frequency and temperature dependencies of the substrate conductivity, with saving results in the file. Then a small amount of metallic atoms is deposited, and again conductivity is measured and saved; again deposition is made an so on. Proceeding by small steps, one should reach a regime, when the film contribution is clearly seen in the substrate background. Then the actual measurements can be made.

2. The second example is a nanocomposite system [9, 10], which is a dielectric sample with the metallic granules embedded in it (Fig.2). The volume fraction p of a metal can be rather large and its effect should be easily observable, being of the order of unity. However, a "hidden rock" is present here. Let exploit the formula from the Landau and Lifshitz book [11]

$$\frac{\bar{\epsilon}-\epsilon_0}{\epsilon_0} = p\,\frac{3(\epsilon_1-\epsilon_0)}{2\epsilon_0+\epsilon_1} \approx 3p - 9p\frac{\epsilon_0}{\epsilon_1}\,, \qquad (1)$$

which is valid for a small concentration of spherical granules: it gives the average permittivity $\bar{\epsilon}$ for the system of Fig.2,a in terms of its values for a dielectric (ϵ_0) and a metal (ϵ_1). Since $\epsilon_1 \gg \epsilon_0$, then the main contribution $3p$ is an uninteresting constant, while the useful effect, depending on ϵ_1, is determining by two small parameters p and ϵ_0/ϵ_1. As a result, the problem of a reference arises, i.e. a necessity to have the identical sample without metallic granules. Fortunately, such a problem is absent for a specific technology [9, 10], when nanocomposites are produced on the base of a porous glass, whose pores are filled by metallic granules; so the same sample can be measured in absence and in presence of granules. It is useful to note that for the system of Fig.2,a (in contrast to that of Fig.1) the strongly disordered metal is desirable, in order to increase the ratio ϵ_0/ϵ_1.

films of the transition metals are usually strongly disordered.

3. Derivation of Eq.1 is based on a solution of a well-known problem on a dielectric ball in the external electric field [11]. The analogous problem is solvable for an ellipsoid with arbitrary ratios of its semi-axes a, b, c [11], and generalization of (1) is possible for granules of ellipsoid form:

$$\frac{\bar{\epsilon}-\epsilon_0}{\epsilon_0} = p\,\frac{\epsilon_1-\epsilon_0}{A\epsilon_0+B\epsilon_1}\,, \qquad (2)$$

where $A = 1 - B$, and

$$B = \frac{abc}{2}\int_0^\infty \frac{dx}{(x+a^2)^{3/2}(x+b^2)^{1/2}(x+c^2)^{1/2}}\,, \qquad (3)$$

if the electric field $\mathbf{E}$ is directed along the axis a.

In reality, the metallic granules are not strictly spherical in the case of Fig.2,a. For modelling of such situation, one can suggest that granules are ellipsoids with fluctuating ratios of semi-axes. Then for $\epsilon_1 \gg \epsilon_0$ one has

$$\frac{\bar{\epsilon}-\epsilon_0}{\epsilon_0} \approx p\left\langle B^{-1}\right\rangle - p\left\langle B^{-2}\right\rangle\frac{\epsilon_0}{\epsilon_1} \qquad (4)$$

($\langle\ldots\rangle$ is averaging over fluctuations), so the structure of Eq.1 is preserved but the coefficients are changed.

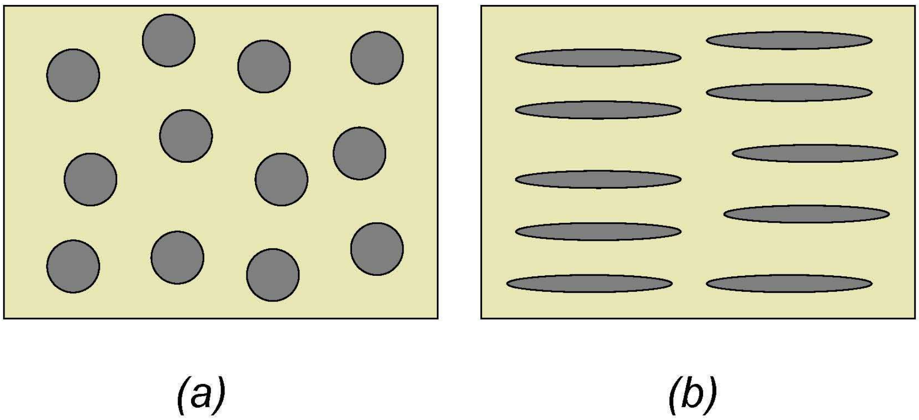

Figure 2: A nanocomposite system with spherical (a) and needle-shaped (b) metallic granules embedded in a dielectric.

Parameter B decreases when a becomes greater than b and c. In the limit of strongly oblong ellipsoid ($a \gg b \sim c$) one has $B \to 0$ and Eq.2 ta

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} = p \, \frac{\epsilon_1 - \epsilon_0}{\epsilon_0} \, ,$$

i.e. optimal conditions for observatio to the needle-shaped granules (Fig.2,b). one can provide a sufficient smallness is necessary for validity of Eq.2 and a interpretation of the experiment) and it tion by a large parameter ϵ_1/ϵ_0. As a is of the order of unity or even more. S can be fabricated on the base of chryso which is a stack of the parallel nanotu a typical pore diameter $5nm$; since the granules should be essentially greater duced to work in the millikelvin range tures.

4. In relation with the latter, let ind otic possibility. If a vessel with supe is rotated, then a set of the parallel vo If metallic atoms are injected in heli localized at the vortex cores and for [14]. Regulating the length of the latter, one can create the desired system (Fig.3). A concentration of the metallic phase is strongly restricted in this case ($p \lesssim 10^{-12}$), but at sufficiently low temperatures one can deal with large L scales and, as a consequence, with enormous values of permeability ϵ_1.

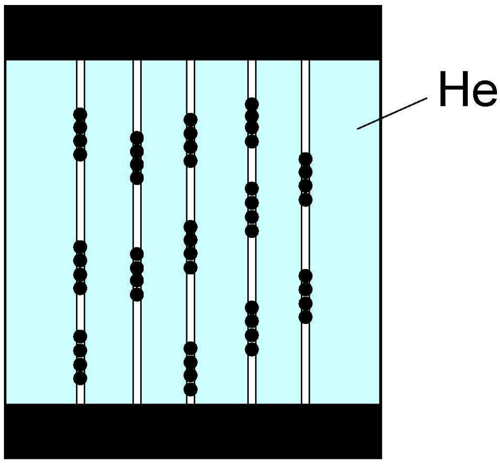

Figure 3: An exotic realization of system represented in Fig.2,b. If a vessel with superfluid helium is rotated, then a set of the parallel vortices arises, and the injected metallic atoms are localized on the vortex cores.

[3] This length can reach $1mm$ [13].

Analogously, parameter B tends to zero in the case of pancake-shaped granules ($a \sim b \gg c$), if their plane is oriented along the electric field; this case is also described by formula (5). In particular, it is valid in the situation of Fig.1, where the volume concentration p is inevitably small.

In conclusion, the Berezinskii law is observable in principle, but the experimental difficulties are present in all considered situations. The latter is rather natural, since in the opposite case this law would be discovered long ago.

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