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Two mechanisms of quantum oscillations of T_c in layered superconducting structures

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Abstract

Quantum oscillations of T_c in layered superconducting structures should be interpreted in terms of two different mechanisms: (a) interference of de Broglie waves on two planar defects and (b) one defect interference confined by the jump of the order parameter on the other defect. The first mechanism leads to nondecaying oscillations, the second to decaying ones.

In Refs. [1–3] the following interesting phenomenon was described: if the superconducting film prepared from material A with the width $L \ll \xi_0$ is covered by material B then the transition temperature T_c oscillates with the change of the width d of the covering material (Fig. 1). The first theoretical consideration of this effect belongs to Kagan and Dubovskii [4]. It was based on the assumption that material B is a normal metal [2] whose electrons do not take part in superconductivity at all, while there is a strong relation between wavefunctions of materials A and B. One can imagine that there is a semipermeable membrane between A and B materials and it transmits normal electrons without transmitting superconducting ones. Then the problem is reduced to calculation of the T_c of the film with

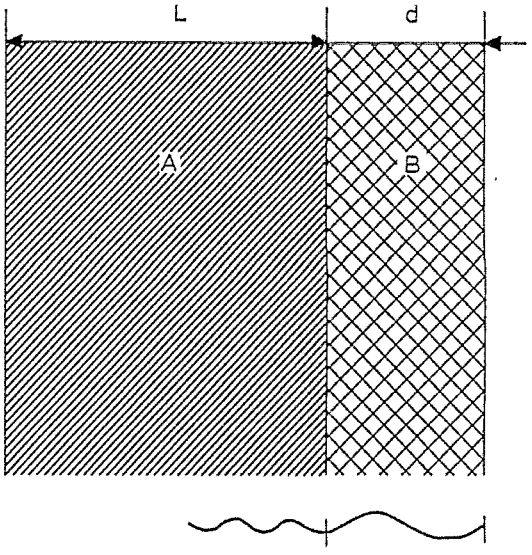

Fig. 1. If superconducting film A is covered by material B then T_c of the system oscillates with changing the width d of the covering material.

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[2] Authors of Ref. [4] called the covering material 'nonmetal' (assuming that it is a semimetal or a semiconductor metallized near the surface due to band bending) stressing the fact that the electron wavelength must be greater then the size of the roughness of the boundary; one does not have to take this limitation into account for an ideally plane boundary.

varying boundary conditions: the phase of the wavefunction has a fixed value on the outer surface of material B and oscillates with the change of d on the boundary between A and B. The critical temperature up to a parameter a/L (a is the interatomic space) depends on the boundary conditions and consequently appears to be an oscillating function of d. It is shown below that Kagan and Dubovskii's consideration requires radical revision and one should interpret the experiments [1–3] in terms of two different mechanisms.

It is easy to see that the concept of a semipermeable boundary [3] is completely unsatisfactory because of the proximity effect: the transverse size of the system $L + d \ll \xi_0$ and one of the partners of the Cooper's pair may penetrate into material B with a probability of the same order as a normal electron may do. Moreover, we are interested only in the transition temperature at which the coupling energy is negligibly small and it is just impossible (without help of Maxwell's demon) to distinguish 'superconducting' and 'normal' electrons. It is obvious that we should consider the A–B sandwich as a unified superconducting system and the concept of varying boundary conditions becomes doubtful. One can try to keep the arguments of Ref. [4] as a reasonable qualitative explanation of the effect, noting that there is a jump of the order parameter on the A–B boundary and the latter can 'work' as an edge of the superconductivity area in Kagan–Dubovskii's picture. But it turns out that this interpretation is also unsatisfactory.

The first argument in this direction follows from Ref. [5]: there are quantum oscillations of T_c even if we remove the difference between materials A and B (and as a consequence the order parameter jump) simultaneously substituting the A–B boundary with a planar defect. The most transparent experimental geometry is presented in Fig. 2(a): in the film of material A two planar defects are inserted, and we can observe the oscillations of T_c with a change of distance d between them. One can give the interpretation of this effect in two limiting cases. In the case of small transparency of the defects we have two

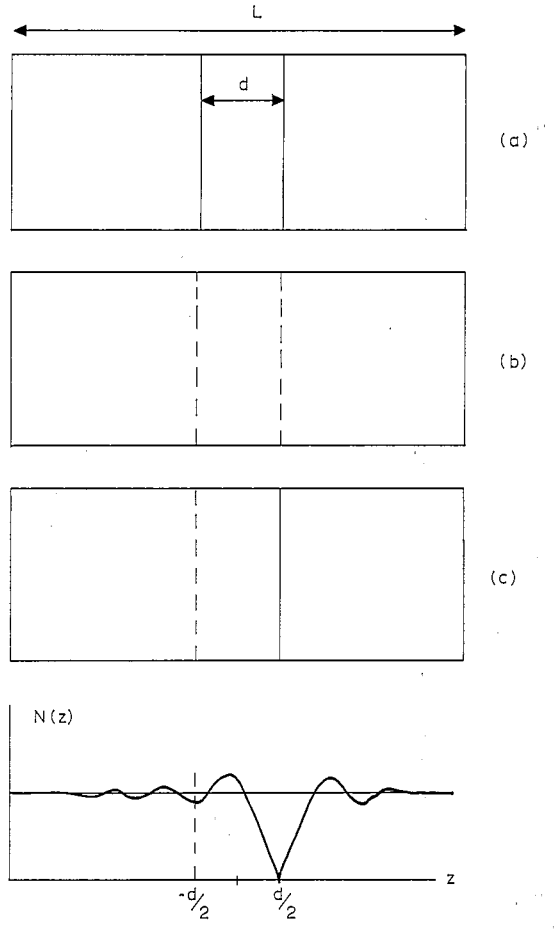

Fig. 2. By changing distance d between two scattering defects (solid lines) one can get nondecaying oscillations of T_c (a); in the case of two unscattering defects (dotted lines) there are no oscillations of T_c (b); in the case of scattering and unscattering defects oscillations of T_c decay (c). Below: local density of states $N(z)$ for the case (c).

weakly coupled systems: a three-dimensional superconductor in the region $|z| > d/2$ (z axis perpendicular to the planar defects and the point $z = 0$ located in the middle between them) and a quasi-two-dimensional system in the region $|z| < d/2$. The spectrum of the last system is the set of 2D bands, the size quantization levels depending on the longtitudinal momentum; with the increase of d, the spacing between the levels decreases and, when the bottom of the next 2D band intersects the Fermi level, there is a jump of T_c due to the jump of the density of states: oscillations of T_c have a saw-like form [5]. In the case of high transparent defects, it is

[3] There is no such image in Ref. [4] but the given interpretation exactly corresponds to the performed calculations.

convenient to consider the electron wavefunctions as a result of repeated reflections of plane waves from defects. The most important is the interference of the incident wave $A\exp(ik_F z)$ (where k_F is the Fermi momentum), and the twice reflected wave $A_1\exp(ik_F z + 2ik_F d)$ (two reflections are necessary to get the information of the existence of two defects, and we can neglect reflections of higher order because the reflection coefficient is small); the result of the interference depends periodically on d which leads to oscillations of T_c.

We can apply this consideration to the system in Fig. 1: the boundary of material B with the vacuum and the interface between A and B materials play the role of planar defects. Thus the existence of the jump of the order parameter is not a necessary condition for the effect of quantum oscillations to take place, but the question of its sufficiency remains: are there oscillations due to the difference of the superconducting properties of A and B only? In order to understand it, let us introduce the concept of an unscattering planar defect: it can be the boundary between two metals M_1 and M_2 with identical spectra and with no surface potential on the interface. If the T_c of M_1 and M_2 are different, then there is a jump of the order parameter on the plane of the defect but there is no scattering of electrons (we can neglect the Andreev reflection because only the vicinity of T_c is considered). The simplest way to create such a defect is to use M_1 metal doping with the impurities damping T_c as M_2. In some approximation, as a defect of this kind, the boundary between the two simple (nontransition) metals may be considered, if they have the same valency, close lattice constants, and work functions. Their spectra are close to $p^2/2m$ with 'bare' electron mass and due to the above conditions they have close Fermi momenta and a small surface potential.

Are there T_c oscillations when we are changing distance d (Fig. 2(b)) between two unscattering defects (denoted by the dotted line)? According to Kagan–Dubovskii's picture, the answer to this question is positive: the phase of the wavefunctions is fixed in the plane $z = 0$ due to the symmetry, and oscillates on the planes of defects when d is changed. As for the mechanism described in Ref. [5], it is artificially 'turned off' – there is no interference because of the absence of scattering. The answer to this question may be given on the basis of the formula for T_c:

$$\frac{\delta T_c}{T_{c0}} = \frac{T_c - T_{c0}}{T_{c0}} = \frac{1}{\lambda_0^3 L}\int \mathrm{d}z\; V_0 N(z)\left[V(z)N(z) - V_0 N_0\right], \tag{1}$$

which follows from Gorkov's equation in the case of local space inhomogeneity [6]. This formula was also used in Ref. [5] [4]; here $V(z)$ and $N(z)$ are, respectively, the coordinate dependent BCS interaction constant and the local density of states; they are equal to V_0 and N_0 outside the region of integration $|z| \lesssim d/2$; $\lambda_0 = V_0 N_0$ is a dimensionless coupling constant, and T_{c0} is the value of T_c corresponding to λ_0. Setting $V(z)$ equal to V_1 in the region $|z| < d/2$ and equal to V_0 in $|z| > d/2$ and taking into account that $N(z) = \mathrm{const} = N_0$ for $|z| \lesssim d/2$ because of absence of scattering, we get

$$\frac{\delta T_c}{T_{c0}} = \frac{1}{\lambda_0}\left(\frac{V_1}{V_0} - 1\right)\frac{d}{L}. \tag{2}$$

So we have only one contribution $\sim d/L$ related to the proximity effect, and there are no oscillations of any kind. Thus oscilations cannot be caused by the order parameter jump only.

Now let us assume that one defect scatters electrons and the other one does not (Fig. 2(c)). As in the previous case, the mechanism of Ref. [5] is not present but full-body oscillations are expected on the basis of Kagan–Dubovskii's picture. In the simplest case of an infinitely strong scattering defect (when zero boundary conditions take place on the plane $z = d/2$) we get [6]:

$$N(z) = N_0\left(1 - \frac{\sin 2k_F z'}{2k_F z'}\right), \quad z' = z - d/2, \tag{3}$$

[4] Eqs. (4.3), (4.4) of Ref. [4] are in agreement with (1) within the assumed model, differing from it only in considering the chemical potential shift (which in reality is absent [6]) and lack of integration over the volume of material B. The latter is a consequence of ignoring the proximity effect and leads to loss of the contribution of order d/L (when effects $\sim a/L$ are under consideration) and the nondecaying component of quantum oscillations (nondecaying oscillations were obtained in Ref. [4] as a result of crude approximations in Eqs. (4.3), (4.4)).

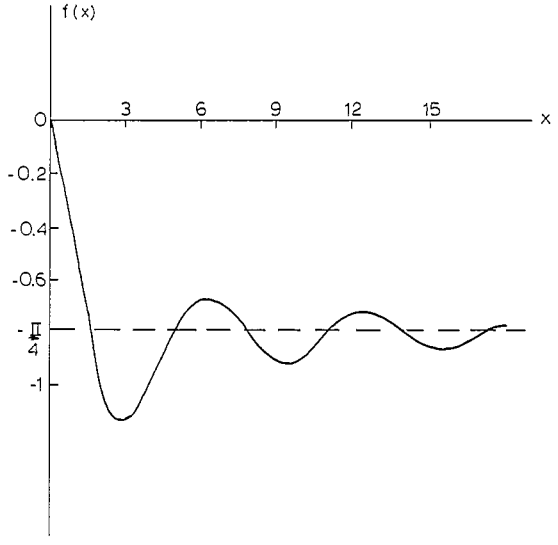

Fig. 3. Function $f(x)$ describes the form of T_c oscillations (Eq. (4)) in the geometry of Fig. 2(c).

and substitution into (1) yields

$$\frac{\delta T_c}{T_{c0}} = \frac{1}{\lambda_0 k_F L}\left(\frac{V_1}{V_0} - 1\right)\left[k_F d + f(2k_F d)\right], \qquad (4)$$

where

$$f(x) = -\mathrm{Si}(x) - \sin^2 x/(2x) + \mathrm{Si}(2x)/2$$

and $\mathrm{Si}(x)$ is the sine integral. It is clear from Fig. 3, that there are oscillations but, contrary to Ref. [4], they are decaying. The mechanism of their appearance is as follows. Far from the scattering defect electrons move freely, i.e. there is a superposition of plane waves $\exp(ikr)$ with different k. At an arbitrary point r, their phases are random and there is no interference:

$$|\sum_k A_k e^{ikr}|^2 = \sum_k |A_k|^2.$$

The value on the right-hand side is independent of r, and is related to the equilibrium electron density. In plane $z = d/2$, we cannot neglect the interference – the phases of the plane waves must be correlated, because we should get zero electron density in accordance with the zero boundary condition. This correlation is decreased while we are remoting from plane $z = d/2$. Thus the presence of a scattering defect leads to the existence of local phase coherency, and there arises an interference picture of decaying intensity. These arguments explain the shape of $N(z)$ (Fig. 2). As the unscattering defect is moved, the shape of $N(z)$ does not change but either the maximum or the minimum of $N(z)$ comes into a 'good' superconductor and oscillations of T_c arises: they decay in accordance with the decay of $N(z)$ oscillations.

Thus we have two different mechanisms of quantum oscillations: the first is due to interference of de Broglie waves on two planar defects, the second is due to interference on one defect which is 'captured' by the jump of the order parameter on the second defect. Correspondingly, it is possible to perform three 'pure' experiments (Fig. 2), in one of them (a) there are oscillations due to the first mechanism, in another (c) – due to the second mechanism; in the third (b) there is no effect because of the absence of the mechanism. It should be stressed that Kagan and Dubovskii's concept of varying boundary conditions leads to wrong results in all three cases. In case (a) it does not predict the existing effect, in case (b) it predicts a non-existing effect, in case (c) it predicts nondecaying oscillations instead of decaying ones. The experiment in geometry of Fig. 2(a) was performed recently [7].

Interestingly, the two described mechanisms are distinctly displayed in the general case, when both scattering and the jump of the order parameter are present. In our extensive investigation of sandwiches A–B–A [8], the following regularity was discovered. If the amplitude of scattering on the interface is proportional to a small parameter ϵ, then the result for T_c has the following form:

$$\frac{\delta T_c}{T_{c0}} = \epsilon(\lambda_1 - \lambda_0) f_1(k_F d) + \epsilon^2 f_2(k_F d), \qquad (5)$$

where $f_1(x)$ is an oscillatory decaying function, and $f_2(x)$ is a oscillatory nondecaying function; their behavior depends on the origin of ϵ (the difference in longtitudinal or transverse masses of A and B, the difference in the position of the band bottom etc.), while the structure of the result (5) is universal. From the above considerations, it is clear that term $\sim \epsilon^2$ is related to the interference on two defects while term $\sim \epsilon(\lambda_1 - \lambda_0)$ is due to the interference on one defect, that is confined by the order parameter jump on the second (within the limits of applicability of Ref. [1], the order parameter is proportional

to $V(z)N(z)$ [6], and its jump is determined by the value of $(\lambda_1 - \lambda_0)$).

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